Multidimensional Uncertainty and Herd Behavior in Financial Markets

Avery and Zemsky (1998), AER

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• Motivation

We do observe price bubbles and financial market crashes

In the sequential trading, we do observe imitative or herd-like behavior.

• Uncertainty

What drives the herding?

What triggers information cascades?

• Financial Market

Can herd behavior lead to long-run mispricing of asset?

Does it produce price bubbles and crashes?
• A simple example (BHW, 1992):

  • Consider a sequence of traders who face a choice of whether or not to adopt a new technology, with the fixed cost of adoption at \( c = 1/2 \)

  • The value of the technology is uncertain, i.e., \( V \) is either 1 or 0.

  • Further, assume that each time one trader arrives at the market and receives a private signal of \( x \), \( x \) can also be either 1 or 0.

  • Assume that \( \Pr(x=1|V=1) = \Pr(x=0|V=0) = p > 1/2 \)

  • Traders do not observe others’ private signal, but the trading history is common knowledge.
• A simple example (BHW, 1992):

  • Now consider a trader arrives at the market and receives \( x = 0 \), but he faces the trading history of (denote the adoption of the technology as 1 while no-adoption as 0):

    \[ (1, 1); (1, 0, 1, 1); (1, 0, 1, 0, 1, 1); \ldots (1, 0, \ldots, 1, 1) \]

  • Intuitively, this trader may think the previous traders received more positive signals than negative ones, and since \( p > 1/2 \), this trader will believe the probability of \( V = 1 \) is higher than 1/2, hence he will also adopt the technology regardless of his private signal \( x = 0 \).

  • After this trader, all the following traders will adopt the technology regardless of their private signals, according to the same logic.
• Why imitative behavior happens in this example?

• Fixed cost: \( c = 1/2 \). Such cost can be interpreted as the price.

• If we make the cost flexible, i.e., \( c' = E[V|H_t] = \Pr(V=1|H_t) \), then all the traders trade according to their private signals.

• Intuitively, whatever the trading history is, the trader who arrives at the market expects the value of the technology conditional on the trading history and his private signal.

• He will adopt iff such expectation exceeds the cost and not adopt iff such expectation is below the cost.

• In this case, however, the cost adjusts according to the trading history too so that the trading history has no impact on the trader’s choice.

• Hence, the trader’s choice is based on his private signal only.
• Glosten and Milgrom (1985)

  • They established a sequential trading model with a competitive market maker (called ‘specialist’ in their paper) who sets a bid and ask price with the interpretation that he is willing to sell one unit of the asset at the ask and buy one unit at the bid.

  • In the market there are informed traders and noise traders. Each time only one trader arrives at the market and he can subscribe one unit of buy order or sell order or leave. The informed traders receive private signal and noise traders trade for exogenous reasons, such as liquidity.

  • Public information is trading history. The market maker can adjust his bid and ask after each trade, according to his own information. He does not know the arrived trader is informed or noise trader.
• Glosten and Milgrom (1985)

• Based on some further assumptions, such as the market maker never regret ex post (i.e., zero profit and recoups the lost from each trade), no transaction cost, etc., they established several beautiful propositions:

• The $\text{Ask} \geq E[V] \geq \text{Bid}$ for each transaction. (Due to adverse selection)

• The sequence of transaction prices forms a martingale. (Due to the competitive market maker and his expectation of the asset value is martingale w.r.p.t. the public information)

• The trading volume times the average bid-ask spread squared is bounded by a number that is independent of the trade pattern.

• The value expectations of the market maker and the informed traders tend to converge, i.e., the insider information tends to be fully disseminated into the market price.
• Avery and Zemsky (1998):

  • Based on Glosten & Milgrom’s setup, Avery and Zemsky studied the possibility of herding in the sequential trading with a market maker.

  • **Value Uncertainty.** if there is only one dimension of such uncertainty of the asset value, herding is impossible.

  • **Event Uncertainty.** if there is also a second dimension of uncertainty, herding is possible but such herding has little effect on asset prices. e.g., uncertainty as whether the asset value has changed from its initial expectation; whether there is a merger.

  • **Composition Uncertainty.** if there is also a third dimension of uncertainty, which means there is uncertainty as to the average accuracy of traders’ information, then price bubbles and financial crashes are possible.
• **Basic setup**

  - Single asset with true value \( V \in [0,1] \).
  - A competitive market maker interacts with an infinite sequence of traders.
  - The sequence of traders is indexed by \( t = 0,1,2,\ldots \)
  - Publicly observable trading history up until time \( t \) is denoted as \( H_t \).

• Informed traders and noise traders:
  - \( \mu < 1 \) be the probability of an informed arriving.
  - Noise traders trade for exogenous reasons, each time when a noise trader arrives, he has the equal probability of buy, sell or leave.
  - Informed traders receive private information \( x_\theta \in [0,1] \)
• **Conditional Expectations and Stochastic Process**

  • The expected value of an informed trader is \( V_\theta^t(x) = E[V \mid H_t, x_\theta = x] \)

  • The market maker’s expected value is \( V_m^t = E[V \mid H_t] \)

  • Minimal amount of “useful” information assumption: if \( |V_m^t - V| = \delta > 0 \)
    then for some \( \varepsilon(\delta) > 0 \), \( |V_\theta^t(x_\theta) - V_m^t| > \varepsilon(\delta) \)

  • With adverse selection, the market maker sets a (bid-ask) spread. With zero profit condition:
    \[
    B^t = E[V \mid h_t = S, H_t] \quad \text{and} \quad A^t = E[V \mid h_t = B, H_t]
    \]

  • Denote \( \pi_v^t = P[V = v \mid H_t] \)
• Information cascade and Herd behavior

  • Information cascade occurs in period \( t \) if:
    \[
    P[h_t \mid V, H_t] = P[h_t \mid H_t]
    \]
    In the information cascade, no new information reaches the market.

  • Herd behavior: the trader with \( x_\theta \) engages in herd behavior at time \( t \) if he buys when \( V_0(x_\theta) < V_m^0 < V_m^t \) or sell when \( V_0(x_\theta) > V_m^0 > V_m^t \)

    • Herd buying: initially he wants to sell, since the signal is pessimistic. However, the trading history is positive such that \( V_m^0 < V_m^t \), and finally the trader gives up his pessimistic signal and join the buy.
• Proposition 1

  • In each period $t$ there exist unique bid and ask prices which satisfy
    $$B_t^{t} \leq V^{t}_m \leq A_t^{t} , \; V^{t}_m \text{ and } \pi^{t}_v \text{ are martingales with respect to } H_t.$$  

  • Comments:
    • the bid-ask spread in proposition 1 is due to adverse selection as in Glosten & Milgrom’s (1985) paper.
    • Market maker expects the value $V_m$ based on the information contained in the prior history of trade. Otherwise, the market maker can make profit of the history information.
• Proposition 2

  • An information cascade never occurs in market equilibrium.

  • Comments:

    • Remember the paper has assumed the minimal useful information property of the private signals, i.e., informed traders are driven by information asymmetries between traders and the market maker.

    • If there is information cascade, then the market maker learns nothing from the trade. Hence $B^t = V_m^t = A^t$. With the private signal, which is “useful”, the informed traders trade according to their private signals. This, however, provide the information to the market maker. A contradiction.
• Before the next proposition

• Definition of *monotonic signal*:

  • A signal $x_\theta$ is monotonic if there exists a function $v(x_\theta)$ s.t. $V_\theta^t(x_\theta)$ is always between $v(x_\theta)$ and $V_m^t$ for all trading histories $H_t$.

• Intuition:

  • If the informed trader receives a signal such that $V_\theta^t(x_\theta) > V_m^t$, then it means, by definition, $v(x_\theta) > V_\theta^t(x_\theta) > V_m^t$, regardless of trading histories.

  • Therefore, such signal would drives the private expectation of the informed trader to the same direction (toward $v(x_\theta)$) for any trading histories.
• Proposition 3

• A trader with a monotonic signal never engages in herd behavior.

• Proof: suppose a trader with a monotonic signal $x_\theta$ engages in herd buying at time $t$. Then it means $V^t_\theta(x_\theta) > A \geq V^t_m$. Hence it means $v(x_\theta) > V^t_\theta(x_\theta) > V^t_m$, hence $V^0_\theta(x_\theta) > V^0_m$. A contradiction.

• Comments:

  • If signal is monotonic, we say there is only one dimension of uncertainty, i.e., value uncertainty.
• **Proposition 4**
  
  • The bid and ask prices converges almost surely to the true value $V$.

• **Corollary 1**
  
  • The variance of price paths is bounded as follows: $\sum_{t=1}^{T} \text{var}(\Delta V_m^t) \leq \text{var}(V)$
  
  • Intuition: the price sequence is a martingale so that $\Delta V_m^t$ is serial uncorrelated. Moreover, the prices converge to the true value.
  
  • Like Glosten & Milgrom, in the sequetial trading with monotonic signals, prices are “stable”. The variance of prices paths is bounded by the underlying uncertainty of the true value of the asset.
• A formal definition

• There is event uncertainty when \( 1 > P(V = V_m^0) > 0 \).

• There is information event if \( V \neq V_m^0 \), i.e., the true value diverges away from the initial belief.

• Assume the informed traders know whether there is information event.

• Information Structure I (IS I)

• The true value \( V \in \{0, 1/2, 1\} \) with initial prior \( \pi_{1/2}^0 > 0, \pi_1^0 = \pi_0^0 > 0 \)

  \( V_m^0 = 1/2 \), and there is event uncertainty.

• There is a single type of informed traders with signal \( x \).
• **Information Structure I (IS I)**

\[
P\left( x = \frac{1}{2} \mid V \right) = \begin{cases} 
1 & \text{if } V = \frac{1}{2} \\
0 & \text{if } V \neq \frac{1}{2}
\end{cases}
\]

\[
P(x = 1 \mid V) = \begin{cases} 
p & \text{if } V = 1 \\
1 - p & \text{if } V = 0
\end{cases}
\]

\[
P(x = 0 \mid V) = \begin{cases} 
p & \text{if } V = 0 \\
1 - p & \text{if } V = 1
\end{cases}
\]

\[
1 \geq p > \frac{1}{2}
\]

• Note that IS I does not satisfy the monotonic signal property because of the value of \(\frac{1}{2}\).
• Proposition 5

• Under IS I, price paths with herd behavior occur with positive probability for $p<1$. They do not occur for $p=1$. Herd behavior is misdirected with positive probability.

• Proof: with noise trader, any trading history can be possible. Suppose in the first $N$ trading history, there is no herding. Fix an $\epsilon>0$. Without herding, each buy order increases the expected value of the asset, and each No Trade increases the probability of $V=1/2$.

• Now an informed trader receives a signal of $x=0$, but he observes $n$ buy orders such that his expected value of the asset is $1/2+\epsilon$.

• Meanwhile, there are $m$ No Trade such that $m+n=N$, and $m>>n$ so that the market maker expects the value of the asset is less than $1/2+\epsilon$, and the ask $A<1/2+\epsilon$. 

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• Proposition 5, contd.

• The difference is that since the informed trader receives the $x=0$, he knows that the information event has occurred, otherwise he would have received $x=1/2$ for sure.

• However, the market maker does not know such information. With the observation of $m$ No Trades, he might interpret them as informed traders with signals of $x=1/2$.

• Proposition 5 suggests that the additional dimension uncertainty dulls the price adjustment in the short run, and the informed traders and the market maker have the different interpretation of the trading history!
• Proposition 6 & 7.

• These two propositions suggest the probability of herd behavior with some exogenous given parameters, which takes time to use probability theory to prove. So I don’t want to introduce them here.

• Proposition 8

• In IS I, during any interval of trading in which there is herd behavior, the movement in the asset price is less than

\[ \Delta = \frac{3\mu(p - 1/2)}{2 + \mu} \]

In the limit as either \( \mu \to 0 \) or \( p \to 1/2 \), we have \( \Delta \to 0 \).
• Proposition 8, contd.

- The formula is not important (unless you want to have the detailed discussion about the stochastic process of trading). What matters is that the price variation is bounded even if with the presence of herding!

- Intuitively, if there are more buy orders than sell orders, and the difference reaches the critical number such that herding begins, the informed traders in the following know there is no new information revealed by the trades during herding, and their valuation of the asset is based on the valuation of the first trader who triggered the herding.

- On the other hand, the market maker takes time to know that the sequential buy orders are due to herding (he might think the herding buying is due to noise traders so that he still believe the information event has not occurred). After collecting enough evidence (too much buy orders), the market maker will finally change his belief and adjust the price.
• Proposition 8, contd.

• However, if the ask price is increased to a level higher than the valuation of the informed trader who first triggered the herding, the herd behavior will stop.

• This is the maximum amount of the price variation.

• Proposition 9

• In IS I, a period of herding reveals more information about the existence of an information event than a period in which agents trade based on their information about value uncertainty.
• Proposition 10

• In IS I, suppose that an information event occurs. In the limit as $\mu \to 0$, the choice of informed traders between herding and trading based on their information about value uncertainty minimizes the deviation of the asset price from its new value.

• Comments:

• In this sense, herding is efficient.

• With the occurrence of information event, herd behavior can change the market maker’s belief faster. E.g., herd buying results that the market maker might consider: are all these buy orders due to noise trader? If no, then the informed traders must have received the signal that is different from $\frac{1}{2}$. 
• **Composition uncertainty**

  • **Definition:** there is composition uncertainty when the probability of traders of different types $\mu_0$ is not common knowledge.

  • Composition uncertainty complicates learning for market participants, especially in the presence of herd behavior.

  • Poorly informed trader $v.s$ well informed trader

  • If one informed trader or the market maker observes a sequence of buy orders, but he does not know other informed traders are poorly informed or well informed.

  • The buying sequence might due to herding for poor informed traders, but it might also due to well informed traders who indeed receive good signals.
• Information Structure II

• The true value \( V \in \{0, 1/2, 1\} \), and all other setups are the same as in IS I except there are two type of traders \( \theta \in \{H, L\} \).

• The difference between the two types is the precision of their information.

\[
P(x_\theta = 1 | V) = \begin{cases} p_\theta & \text{if } V = 1 \\ 1 - p_\theta & \text{if } V = 0 \end{cases}
\]

\[
P(x_\theta = 0 | V) = \begin{cases} p_\theta & \text{if } V = 0 \\ 1 - p_\theta & \text{if } V = 1 \end{cases}
\]

- Hence the \( H \) type traders are perfectly informed.
• Information Structure II

• There can be a lot of $H$ type traders or a lot of $L$ type traders in the market.

• Index $I \in \{W, P\}$ denotes the market to be Well or Poorly informed.

• Let $\mu_{\theta}^I$ be the probability of a type $\theta$ trader in a type $I$ market.

• Assume $\mu_H^I + \mu_L^I \equiv \mu$ hence there is a fixed probability of an informed trader in type $I$ market.

• Naturally assume $\mu_W^I > \mu_P^I$
• Simulation

• Let $\pi_{1/2}^0 = 0.9999$ such that the market maker would believe the information event can occur with little probability.

• Let $p_L = 0.51$ such that the $L$ type traders are really poorly informed.

• Let $\mu_H^P = 0$ such that there is no $H$ type traders in the Poor market. Hence there is very little information about value uncertainty in any trade in the Poor market.

• Other variables: $\gamma = 0.25$, $\mu_L^W = \mu_H^W = 0.125$, $\mu_L^P = 0.25$

• The true state is $(V, I) = (0, P)$ and last but not least $\pi_{v,W}^0 / \pi_{v,P}^0 = 99$ such that everybody believes that a Well informed economy is highly more likely!
• Price bubble
• Trading history
• One dimension of uncertainty
  • If there is only value uncertainty and the signal is monotonic, herding will never occur, because the bid-ask prices reveal the information.

• Two dimensions of uncertainty
  • If there is an additional uncertainty which makes the signal non-monotonic, herding is possible but the price paths are bounded because if herding happens, every informed trader knows.

• Three dimensions of uncertainty
  • If a third uncertainty enters the economy, which makes both market maker and informed traders unsure about whether herding has happened, then asset prices can be extremely volatile which lead bubbles and crashes.