Macroeconomics (Research, WS10/11)
Problem Set 5

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1. Barro-Gordon model

As Barro and Gordon (1983a, b), assume a social loss function depending on employment l and prices p

$$L = (l - l^*)^2 + \beta(p - p^*)^2,$$

where l^* is efficient employment and p^* is the price level consistent with optimal inflation. All lower case letters denote logarithmic terms. The short-run Phillips curve is given by

$$l = \bar{l} + c(p - p^e + \theta),$$

where $c > 0$ is a parameter and θ is a random shock.

a) Assume that the central bank can control the price level and aims at minimizing social losses after observing productivity shock θ. Derive the first order condition for optimal monetary policy and solve the model for its rational expectations equilibrium described by $p^e = E(p)$ and policy rule $p(\theta)$.

b) Discuss the impact of exogenous parameters on the inflation bias $p^e - p^*$ and on the policy rule $p(\theta)$ obtained in **b)**.

c) Assume now that the central bank commits to stabilize inflation in such a way that $p = p^*$. Compare the resulting variance of employment, the inflation bias and expected welfare loss with your solution from **b)**.

☞ Blanchard and Fischer (1989), Chapter 11.4. *Or*

☞ Illing (1998).

2. Solving time-inconsistency problem: Delegation

(Rogoff, 1985) Consider an Economy in which efficient employment and optimal price level are both normalized to 1, $L^* = 1 > \overline{L}$, $P^* = 1$ and $\overline{L}$ is the natural rate of employment. For simplicity, in the following we use log values of variables; therefore $l^* = \ln L^* = 0$, $p^* = \ln P^* = 0$, and $l = \ln L$, $p = \ln P$ are the percentage deviations from their efficient levels.

Suppose the government wants to maximize the social welfare as given by

$$W = \gamma l - a\frac{p^2}{2},$$

and delegates monetary policy to a central banker who follows an objective function

$$\tilde{W} = c\gamma l - a\frac{p^2}{2},$$

in which γ is a random variable with mean $\overline{\gamma}$ and variance σ_γ^2. Suppose that the short-run Phillips curve is given by

$$l = \bar{l} + b\,(p - p^e)\,.$$

Note that the expected price level p^e is determined before γ is observed, and the central banker chooses p after γ is known.

a) Compute the central banker's optimal solution for p, with p^e, γ and c being given.

b) Is the central banker able to resist the temptation to aim at efficient employment, i.e. $l^* = 0$? Compute p^e.

c) Compute the expected value of W.

d) Compute c that maximizes W. Provide some intuitions on your result.

☞ Illing (1998).

3. Solving time-inconsistency problem: Reputation

(Cukierman and Meltzer, 1986) Consider that a monetary policy maker has a limited tenure for only two periods. The policy maker is randomly nominated from a pool of candidates, whose object function is as following

$$W = E\left[b(p_1 - p_1^e) + cp_1 - \frac{ap_1^2}{2} + b(p_2 - p_2^e) + cp_2 - \frac{ap_2^2}{2}\right]$$

in which c is normally distributed over the candidates with mean $\overline{c}$ and variance $\sigma_c^2 > 0$. However, a and b are the same for all candidates.

The policy maker only has a limited control over inflation such that $p_t = \hat{p}_t + \epsilon_t$, $t \in \{1, 2\}$, in which $\hat{p}_t$ is the policy chosen by the policy maker with p_t^e being given and ϵ_t is a normally distributed random variable with mean zero and variance $\sigma_\epsilon^2 > 0$. The random variables, ϵ_1, ϵ_2 and c are independent on each other. The public cannot observe $\hat{p}_t$ or ϵ_t, but only p_t. The public cannot observe c, either.

The public's expectation on the second-period price level, p_2^e, is formed on the basis of observed first-period price level p_1 in a way such that

$$p_2^e = \alpha + \beta p_1.$$

a) What is the policy maker's choice on $\hat{p}_2$? Compute the expected value of her second-period objective function in terms of p_2^e.

b) What is the policy maker's choice on $\hat{p}_1$, with α and β being given and taking account of the impact of p_1 on p_2^e?

c) Compute the proper value of β. Provide some intuitions on your result.

d) Provide some intuitions on why the policy maker chooses a lower $\hat{p}$ in the first period than in the second.

4. Monetary policy: Limited control and incomplete information

Suppose the central bank wants to minimize a welfare function

$$L = E\left[(\pi - \pi^*)^2\right],$$

where π^* is the optimal inflation rate. The central bank has no direct control over the price level. The inflation rate is given by

$$\pi = \rho Z + \eta,$$

where Z is the instrument to the disposal of the central bank, $\rho > 0$ is some parameter and η is a random term with standard normal distribution.

a) What is the optimal reaction of the central bank to shocks η?

b) Suppose now that the central bank cannot observe η but only some variable $\Psi = \zeta + \eta$, where $\zeta \sim N\left(0, \sigma^2\right)$ and ζ and η are independent. What is the optimal response to observed shocks Ψ in this case?

c) Suppose now that the central bank can observe η, but not ρ, which has a normal distribution with mean $\overline{\rho}$ and variance τ^2 . What is the optimal response of the central bank to observed shocks η?

☞Blanchard and Fischer (1989), Chapter 11.4.

5. Monetary policy: Interest targeting versus monetary targeting

Suppose the economy is described by linear IS and LM curves that are subject to disturbances

$$y = c - ai + \epsilon, \text{ and } m - p = hy - ki + \eta,$$

where a, h, and k are positive parameters and ϵ and η are independent mean zero shocks with finite variances. The central bank wants to stabilize output, but cannot observe y or the shocks ϵ and η. Other variables are observable. Assume for simplicity that p is fixed.

a) What is the variance of y if the central bank fixes the interest rate at some level $\bar{i}$?

b) What is the variance of y if the central bank fixes the money supply rate at some level $\overline{m}$?

c) Under which conditions does interest targeting lead to a lower variance of output than monetary targeting?

d) Describe the optimal monetary policy, when there are only IS shocks (the variance of η is zero). Does money or interest rate targeting lead to a lower variance of y?

e) Describe the optimal monetary policy, when there are only LM shocks (the variance of ϵ is zero). Does money or interest rate targeting lead to a lower variance of y?

f) Provide some intuitions on your results from **d)** and **e)**.

g) When there are only IS shocks, is there a policy that produces a variance of y that is lower than either money or interest rate targeting? If so, what policy minimizes the variance of y? If not, why not?

☞Romer (2006), Chapter 10.6.

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