

Macroeconomics (Research, WS10/11)

Problem Set 4

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1. Seigniorage and inflation

Seigniorage, which is the real revenue the government obtains from printing new currency, and exchanging it for real goods, is defined à la Cagan (1956),

$$S = \frac{M_t - M_{t-1}}{P_t}$$

where M_t is the money supply at t and P_t the price level at t . Assume that the demand for real money balances is given by

$$\frac{M_t}{P_t} = \left(\frac{P_{t+1}}{P_t} \right)^{-\eta}$$

with $\eta > 0$.

a) Give some interpretation to the money demand function.

b) Assume the government controls the growth rate of money supply $\frac{M_t}{M_{t-1}} = 1 + \mu$. Show that, correspondingly, inflation will be constant μ all the time. If the government tries to maximize its revenue, what is the optimal μ ? Provide some interpretation to your solution.

c) Compute the loss in consumer surplus and the deadweight loss arising with this optimal μ .

📖 Blanchard and Fischer (1989), CHAPTER 4.7 SEIGNIORAGE AND INFLATION.

2. Money in the utility: The steady state

Consider an infinitely lived agent with utility function

$$\int_0^{+\infty} [c(t) + V(m(t))] e^{-\rho t} dt,$$

where c is consumption, m are real money holdings, and V is an increasing and concave function. Money is the only asset, yielding a nominal interest rate flow $i(t)$. Income is exogenously given by $y(t)$.

- a) Formulate the transition equation in real balances (money holdings).
- b) Formulate the Hamiltonian and first order conditions.
- c) The growth rate of nominal money supply is given by μ . Derive a differential equation describing the optimal real balances.
- d) Discuss potential steady state equilibria and their stability. Characterize conditions that rule out hyperinflationary bubbles.
- e) Discuss the special case of $V(m) = m^\alpha$.

📖 Walsh (2010), CHAPTER 2.1, 2.2.

3. Money in the utility: The dual form

Consider a discrete version of Sidrauski's money in the utility approach: An infinitely lived representative agent maximizes discounted life time utility

$$\sum_{t=0}^{+\infty} \beta^t U(c_t, m_t)$$

with $\beta \in (0, 1)$ as discount rate, c_t consumption and $m_t = \frac{M_t}{P_t}$ as real money balances. Each period, the agent is endowed with y_t . y_t can be used for private or government consumption: $y_t = c_t + g_t$. Initially, the agent owns the money stock M_0 and one period nominal government bonds B_0 . Period t bonds B_t yield a return i_t . The government finances g_t via taxes τ_t , seigniorage or government bonds. To make it easier, assume that there is no nominal return for holding money, i.e. $i_{m,t} = 0$; also the endowment economy implies that there's no other investment opportunity except holding government bonds.

- a) Formulate the period budget constraint of both the agent and the government and derive the present value budget constraint. To make the computation easier, you may define an auxiliary state variable, *beginning of period t wealth* of either party, as $W_t = (1 + i_{t-1})B_{t-1} + M_{t-1}$.
- b) Characterize the first-order conditions for the agent's optimal path.

c) Show that with additive separable preferences $U(c_t, m_t) = u(c_t) + v(m_t)$, the real rate of interest depends only on the time path of the real resources available for consumption.

d) Assume that $U(c_t, m_t) = c_t^\alpha + m_t^\alpha$. Derive the money demand function $m(c_t, i_t)$ and characterize elasticity with respect to c_t and i_t . Show why the price level may not be determinate if the central bank pegs the interest rate to a fixed level $i_t = \bar{i}$.

e) Assume that both endowment and government spending are constant: $y_t = y$; $g_t = g$. Characterize conditions for steady state. Show that the Friedman rule maximizes per period utility. Discuss reasons why this rule may not be optimal in a more general setting.

📖 Walsh (2010), CHAPTER 2.1, 2.2, 2.4.

4. Dixit-Stiglitz indices for continuous commodity space

(Dixit and Stiglitz, 1977) Consider a one-person economy. Mr. Robinson Crusoe is the only agent in this economy, consuming a continuum of commodities $i \in [0, 1]$. Suppose that the consumption index C of him is defined as

$$C = \left[\int_0^1 Z_i^{\frac{1}{\eta}} C_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}$$

in which C_i is the consumption of good i and Z_i is the taste shock for good i . Suppose that Crusoe has an amount of endowment Y to spend on goods with exogenously given price tags. Therefore the budget constraint is

$$\int_0^1 P_i C_i di = Y.$$

a) Find the first-order condition for the problem of maximizing C subject to the budget constrain. Solve for C_i in terms of Z_i , P_i and the Lagrange multiplier on the budget constraint.

b) Use the budget constraint to find C_i in terms of Z_i , P_i and Y .

c) Insert the result of b) into the expression for C and show that $C = \frac{Y}{P}$, in which

$$P = \left(\int_0^1 Z_i P_i^{1-\eta} di \right)^{\frac{1}{1-\eta}}.$$

d) Use the results in b) and c) to show that

$$C_i = Z_i \left(\frac{P_i}{P} \right)^{-\eta} \left(\frac{Y}{P} \right).$$

Interpret this result.

5. Price setting with differentiated goods

Consider a representative agent with utility function

$$U = \left(\sum_{i=1}^m C_i^\gamma \right)^{\frac{\alpha}{\gamma}} \left(\frac{M}{P} \right)^{1-\alpha} - N^\beta, \text{ with } 0 < \gamma < 1, 0 < \alpha < 1, \beta > 1.$$

Assume that firm's profits are distributed to consumers, but a single consumer's decision has no impact on these profits. Thus, profit income is taken as exogenous by consumers.

a) Derive the demand functions for commodities C_i and for money M and the supply for labor N . To ease your calculations, use aggregate indices for consumption and prices:

$$C = \left(\sum_{i=1}^m C_i^\gamma \right)^{\frac{1}{\gamma}}, P = \left(\sum_{i=1}^m P_i^{-\frac{\gamma}{1-\gamma}} \right)^{-\frac{1-\gamma}{\gamma}}.$$

b) Assume that firms produce goods with production function $C_i = \theta N_i$, where N_i is the labor input of firm i . Labor is homogeneous and the labor market is competitive. Firms are setting prices P_i in order to maximize profits. Show that equilibrium prices are above marginal costs (Assume that firms are small to the extent that a single firm's decisions has no impact on average income of households).

c) Show that equilibrium levels of production and employment are below the efficient levels.

d) Following Blanchard & Kiyotaki (1987), explain how menu costs can prevent price adjustments to monetary expansion and how this influences overall efficiency.

📖 Blanchard and Fischer (1989), CHAPTER 8.1.

📖 Blanchard and Kiyotaki (1987).

📖 Romer (2006), CHAPTER 6.4¹.

¹This is by far the smallest (and surely the simplest) model in the world integrating imperfect competition and price setting decisions. If you find it difficult to go through Blanchard and Fischer (1989), CHAPTER 8.1 and Blanchard and Kiyotaki (1987), start from here.

References

- BLANCHARD, O. AND S. FISCHER (1989):** *Lectures on Macroeconomics*. Cambridge: MIT Press.
- BLANCHARD, O. J. AND N. KİYOTAKI (1987):** “Monopolistic Competition and the Effects of Aggregate Demand.” *American Economic Review*, 77, September, 647–666.
- CAGAN, P. (1956):** “The Monetary Dynamics of Hyperinflation.” In Friedman, M. (Ed.) *Studies in the Quantity Theory of Money*. Chicago: University of Chicago Press.
- DIXIT, A. K. AND J. E. STIGLITZ (1977):** “Monopolistic Competition and Optimal Product Diversity.” *American Economic Review*, 67, June, 297–308.
- ROMER, D. (2006):** *Advanced Macroeconomics (3rd Ed.)*. Boston: McGraw-Hill Irwin.
- WALSH, C. E. (2010):** *Monetary Theory and Policy (3rd Ed.)*. Cambridge: MIT Press.