Macroeconomics (Research, WS10/11)

Sample Exam

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Instructions

(1) This is a closed book exam. It is only allowed to use non-programmable calculators.

(2) All your answers should be written on the stamped exam sheets. As for the *Studiengang* in the front page, please specify which of the following programs you belong to: *Ph.D.*, *Master / Bachelor at LMU*, or *exchange program*.

(3) You are expected to answer all parts of all questions. If you cannot solve part of a question, do not give up. The exam is designed so that you should be able to answer later parts even if you are stuck by earlier parts.

(4) The number attached to each problem shows the maximum points you can get from this problem. The time limit for the exam is 120 minutes, and the maximum total score is 120. You may use this information to allocate your time for each problem.

Good luck!

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1. Shorter questions (30 points)

a) One company's share price can be a good proxy for its Tobin's q. To make our discussions easier, assume that there's no bubble the investors have perfect information about the listed firms so that the share prices are close to their fundamental values.

Once Microsoft was proposed to be split up into a couple of "Baby Bill"companies, each of which owning a copy of Windows and Office programs. Asked about whether Microsoft shareholders will benefit from such proposal, the Chief Executive, Steve Ballmer, responded:

> "*To split up the company and let several competitors sell similar products would drive the price of the software so low that neither of them could make a profit. . . . I was an economics major, and learned enough of economics to know that if you have several guys selling the same thing and the marginal cost is zero, the price on that is a well-known economic fact. And yet you have people say that it will increase share price . . . I would be strongly against any notion there would be enhanced shareholder value on breakup.*"

Using the theory of investment (Tobin's q theory), discuss whether Steve Ballmer's argument is consistent with Microsoft's claim in many courts (the most recent one is against the European Commission) that it is not a monopoly.

b) Grilli, Masciandaro, Tabellini (1991) defined an independence index of a central bank, which is computed from the following factors: (1) Central bank governor not appointed by the government; (2) Central bank governor's tenure longer than 5 years; (3) All the central bank's Board not appointed by the government; (4) No government approval of monetary policy formulation is required; (5) No mandatory participation of government representative in the Board; (6) Legal provisions that strengthen the central bank's position in conflicts with the government are present. Then the authors found that the higher one central bank's index is, the lower the country's inflation.

In the framework of Barro-Gordon model, provide some intuitions why central bank independence may help to combat inflation.

2. Regret minimization and economic growth (45 points)

This section is an extension of the standard decentralized Ramsey-Cass-Koopmans model.

Some early economists (including Ramsey himself) disagree with the notion of discounting, i.e. they insist that the discounting rate ρ should be zero and the object function of representative agent with neoclassical utility function $u(c_t)$ be

$$\max_{\{c_t\}_{t=0}^{+\infty}} \int_0^{+\infty} e^{-\rho t} u(c_t) dt = \int_0^{+\infty} u(c_t) dt. \tag{1}$$

Since the object function in (1) is not bounded under $\rho = 0$ and thus this optimization problem cannot be directly solved, an alternative approach is proposed as following: Define the long run upper limit of the agent's utility as *bliss point B* such that

$$B = \max_{\{c_t\}_{t=0}^{+\infty}} u(c_\infty),$$

i.e. for all feasible consumption paths $\{c_t\}_{t=0}^{+\infty}$, B is the maximal utility one can achieve in the long run (or, the steady state utility level), and then the original problem (1) is equivalent to minimizing the representative agent's *regret R* on her infinite life horizon

$$\min_{\{c_t\}_{t=0}^{+\infty}} R = \int_0^{+\infty} [B - u(c_t)] \, dt, \tag{2}$$

that is, R captures the welfare loss arising from gap between the instantaneous utility level along the growth path and the long run optimum B, and the optimal growth path should converge to B as fast as possible to minimize the social regret.

Her flow budget constraint of asset holdings a_t is given by

$$\dot{a}_t = w_t - c_t + r_t a_t \tag{3}$$

in which w_t and r_t are wage and real interest rate for competitive labor / capital market, such that

$$\begin{aligned} r_t &= f'(k_t) - \delta, \\ w_t &= f(k_t) - k_t f'(k_t) \end{aligned}$$

where $f(k_t)$ is the neoclassical production function, and $0 < \delta < 1$ is the constant depreciation rate. There's neither population growth nor technological progress.

Intriligator (1971) has already shown that, with given parameter values and the initial state, the bliss point B is a constant in this model (i.e. you can treat it as a constant in your computation as well), and the object function in (2) is bounded. Therefore the optimization problem can be solved. To ease your computation, assume that $u(\cdot)$ is a CRRA utility function, $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$.

a) Consider the optimization problem defined by (2) and (3). Using Hamiltonian, derive the

Euler equation for the representative agent (Hint: Minimizing R is equivalent to maximizing $-R$.).

b) Using phase diagram in $c-k$ space, illustrate the system dynamics in each subspace. Show graphically (without formal proof) that there exists a saddle path towards the equilibrium.

c) What is the golden rule capital intensity k_g suggested by this model? What's the difference between k_g here and the one from the standard partial equilibrium Solow-Swan model with neither population growth nor technological progress? Provide some interpretation on your findings.

d) Suppose that the economy is already in its steady state. Now news comes that a permanent tax on capital-income-before-depreciation is to be introduced *in one year*, i.e. in the future $\tau f'(k_t)$ $(0 < \tau < 1)$ will be levied as tax (the real interest rate will become $r_t = (1-\tau)f'(k_t) - \delta$ then), and the tax revenue will be redistributed through lump sum transfers. Using the phase diagram, illustrate the economy's transition path from now on. Then explain how the real interest rate responds immediately after the policy is announced, and how it changes along the transition path.

3. Cost of inflation and optimal monetary policy (45 points)

The canonical theory of monetary policy assumes that inflation is costly for society. In the following we explore a justification for that assumption, using Sidrauski's money-in-the-utility function.

Consider an economy with a representative infinitely lived consumer whose preference is given by

$$\sum_{t=0}^{+\infty} \beta^t \left[u(c_t) + v(m_t)\right] \tag{4}$$

in which $u(\cdot)$ and $v(\cdot)$ are increasing and strictly concave utility functions, c_t is consumption at date t, m_t is real money balance at the end of period t, and $\beta \in (0, 1)$ is the discount factor.

Let b_t be real bond holdings at the end of period t that pay a nominal interest rate i_{t+1} at the beginning of the next period, $M_t = P_t m_t$ be nominal money balances, P_t be the price level, and y be the time-invariant and exogenous real income received by the consumer each period. The consumer also receives real net transfers from the government, τ_t. Then, to ease your computation, the nominal budget constraint of the consumer is given by

$$P_{t-1}b_{t-1}(1 + i_t) + P_t y + P_t \tau_t = P_t c_t + M_t - M_{t-1} + P_t b_t.$$

a) Let r_t be the real interest rate and π_t be the inflation rate, such that

$$1 + i_t = (1 + r_t)(1 + \pi_t).$$

Show that consumption can be written as

$$c_t = b_{t-1}(1 + r_t) + y + \tau_t - m_t + \frac{m_{t-1}}{1 + \pi_t} - b_t. \tag{5}$$

b) Using (4) and (5), show that the following efficiency conditions hold

$$-u'(c_t) + \beta \frac{u'(c_{t+1})}{1 + \pi_{t+1}} + v'(m_t) = 0, \tag{6}$$
$$-u'(c_t) + \beta u'(c_{t+1})(1 + r_{t+1}) = 0. \tag{7}$$

Provide some intuitions for equations (6) and (7), and show that (6) and (7) define a money demand function

$$v'(m_t) = \frac{i_{t+1}}{1 + i_{t+1}} u'(c_t). \tag{8}$$

What is the relationship between money demand and nominal interest rates for a given level of consumption? What is the relationship between money demand and consumption for a given nominal interest rate?

c) Assume, for the rest of the problem, that there is no government expenditure and no public debt, so that government prints money only to make net transfers to the consumer, i.e.

$$M_t - M_{t-1} = P_t \tau_t.$$

Since there is only one consumer and the government does not issue public debt, equilibrium requires $b_t = 0$. What is c_t in equilibrium? Using your expression for c_t and equation (7), derive the expression for the real interest rate.

d) Assume, in addition, that the government follows a constant nominal money growth rule

$$M_t = (1 + \mu) M_{t-1}. \tag{9}$$

Define a steady state in this model as a situation in which real variables do not change. In particular, in the steady state $m_t = \overline{m}$. Given (9) and the fact that $m_t = \frac{M_t}{P_t}$, find the steady-state level of inflation in this model, call it $\overline{\pi}$.

e) Using equation (8) evaluated in steady state, find an expression for $\overline{m}$ in terms of $\overline{\pi}$. What is the steady state effect of $\overline{\pi}$ on $\overline{m}$? What is the effect of $\overline{\pi}$ on steady-state consumption? What is the welfare effect of increasing $\overline{\pi}$? What is the optimal level of steady-state inflation?

References

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