

Illiquidity, insolvency, and banking regulation ★

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Abstract

This paper provides a compact framework for banking regulation analysis in the presence of uncertainty between systemic liquidity and solvency shocks. It is shown that systemic liquidity shortage arises endogenously as part of the inferior mixed strategy equilibrium. The paper compares different traditional regulatory policies which intend to fix the inefficiencies, and shows how the co-existence of illiquidity and insolvency problems adds extra cost for banking regulation and makes some schemes that are optimal under pure illiquidity risks (such as liquidity regulation with lender of last resort policy) fail. A banking tax is proposed to cover the extra regulatory cost. The regulatory cost can also be reduced by combining the advantages of several instruments.

JEL classification: E5, G21, G28

Key words: liquidity risk, insolvency risk, liquidity regulation, equity requirement

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1 Introduction

In the banking literature, illiquidity and insolvency problems have been intensively studied for decades. Illiquidity means that one financial institution is not able to meet its short term liability via monetizing the future gains from its long term projects — in other words, there's a mismatch between the time when the long term projects return and the time when its liability is due, i.e., it's "cash flow trapped" but "balance sheet solvent." In contrast, insolvency of a financial institution generally means that liabilities exceed assets in its balance sheet, i.e., it is not able to meet due liabilities even by perfectly monetizing the future gains from its long term projects. Existing banking models usually focus on either problem. If a financial firm's ailment is diagnosed to be one of them, the solution is (at least intuitively) clear. For example, illiquid banks may be bailed out by central bank's liquidity injection (against their illiquid assets "good" collateral, see Cao & Illing, 2010a, b), and insolvent banks have to be closed down in order to avoid contagion (see Freixas, Parigi and Rochet, 2004).

Since mid-2007, the world has seen one of the worst financial crises in history. One of the most remarkable features about this crisis is the ambiguity in the financial institutions' health, especially the daunting question whether the problem for the large banks is illiquidity or insolvency. Financial innovation in the past two decades doesn't only help to improve market efficiency, but it also creates high complexity (hence, asymmetric information) which blurs the boundary between illiquidity and insolvency. The sophisticated financial products, as Gorton (2009) states, finally "could not be penetrated by most investors or counterparties in the financial system to determine the location and size of the risks." For example, subprime mortgages, the financial innovation triggering the current crisis, were designed to finance riskier long-term borrowers via short-term funding. So when the trend of continuing US house price appreciation started to stagger and giant investment banks ran into trouble, the trouble seemed to be a mere illiquidity problem — as long as house prices were to increase in the future, the long-term yields of subprime mortgage-related assets would be juicy, too. However, since the location and size of the risks in these complicated financial products could not be fully perceived

even by the designer banks themselves, there was a probability that these financial institutions were actually insolvent. In this vague scenario banks could hardly get sufficient liquidity from the market and the crisis erupted.

These events bring new challenges to both market practitioners and banking regulators. If there's no ambiguity between illiquidity and insolvency, conventional wisdom works well: if the problem is just illiquidity, liquidity regulation works perfectly — banks can get enough liquidity from the central bank with their long-term assets as collateral, since the high yields from these assets will return in the future with certainty. If the problem is insolvency, equity holding can be a self-sufficient solution for the banks to eliminate their losses. However, if there's uncertainty about the nature of the banks' trouble, things become complicated — banks cannot get enough liquidity because the collateral, in the presence of insolvency risk, is no longer considered to be good. Therefore, liquidity regulation may fail. On the other hand, equity requirements may be inefficient as well because the co-existence of the two problems make equity holding even costlier. This paper tries to shed some light on understanding the market failure and designing proper regulatory rules with a compact and flexible model.

1.1 Summary of the paper

In this paper, banks are intermediaries financing entrepreneurs' short-term (safe) and long-term (risky) projects via short-term deposit contracts, as in Diamond & Rajan (2006). Illiquidity is modelled as Cao & Illing (2010b): some fraction of risky projects turns out to be realized late. The aggregate exposure to the risks is endogenous; it depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity captures the feedback from liquidity provision to risk taking incentives of financial intermediaries.

Unlike in models with pure illiquidity or insolvency problems, market participants only observe the aggregate amount of early returns from the risky projects in the intermediate period. However, they don't know whether these risky assets are just illiquid (i.e., the majority of high yield risky projects will return late), or

whether the banks are insolvent (i.e., a substantial amount of the risky projects will fail in the next period). The introduction of such ambiguity has both significant impact on equilibrium outcomes and new implications for banking regulation.

Given the same structure of the banking model as in Cao & Illing (2008, 2010b), the equilibria in this extended model are similar: two types of pure strategy equilibria — the banks coordinate to be risky when the sun always shines and be prudent when it always rains, and a mixed strategy equilibrium for an intermediate probability of having good luck. However, the gap between the expected return from the risky projects in the good state and that in the bad state gets larger with the uncertainty on the true problem — asset price is more inflated in the good state, while it is more depressed in the bad state. The bigger gap makes the interval for mixed strategy equilibrium wider in current setting, making free-riding more attractive (more excessive liquidity supply when time is good).

New insights have been derived for banking regulation. The solution for the pure illiquidity risk case, as proposed in Cao & Illing (2010a), is to have *ex ante* liquidity requirements with *ex post* conditional bailout. This is not sufficient now. Because the central bank doesn't have superior knowledge compared to market participants, i.e., it isn't able to distinguish between illiquidity and insolvency risks, the value of the banks' collateral in the bad state cannot be as high as that at that in the good state. Therefore, the banks cannot get sufficient liquidity from the central bank in the bad state even if they do observe the *ex ante* liquidity requirement. A costly bank run can thus no longer be avoided.

This finding suggests that the additional insolvency risk implies an extra cost for stabilizing the financial system, i.e., the regulator needs extra resources to hedge against the insolvency risk. Therefore, a counter-cyclical deposit insurance mechanism will work. The proposal is as follows: the banks have to be taxed away part of their revenue in the good state and the taxation revenue can be used to cover the cost in central bank's liquidity provision in the bad state.

It is worth mentioning equity requirements, as the typical solution in the case of pure insolvency risk, is suboptimal as well. The co-existence of two banking

plagues means higher equity ratio, hence higher cost, should be imposed for banking industry.

Since it's hard to catch two rabbits at the same time, it might be optimal to combine the advantages of several instruments. A hybrid regulatory scheme is therefore proposed in this paper, allowing liquidity regulation to discourage the inferior mixed strategy equilibrium (which leads to liquidity shortage) and equity requirement to absorb the loss from insolvency.

1.2 *Review of literature*

This paper is an extension of Cao & Illing (2008, 2010a, b). There, it has already been shown that with only pure illiquidity risk, there's an incentive for a financial institution to free-ride on liquidity provision from the others, resulting in excessively low liquidity in bad states. Since illiquidity is the only risk, conditional (with *ex ante* liquidity requirements for banks' entry to the financial market), a liquidity injection from the central bank fully eliminates the risk of bank runs when bad states are less likely. The outcome of such conditional bailout policy dominates that of equity requirements since the banks have to incur a high cost of holding equity in order to fully stabilize the system. However, when insolvency is mixed with illiquidity and market participants cannot distinguish between the two, banks will have difficulties in raising sufficient liquidity using their assets as collateral. This may have profound impacts on both equilibrium outcomes and policy implications. Exploring these issues is the main task of this paper.

Although illiquidity and insolvency problems independently have been intensively studied in the banking literature, the endogenous systemic liquidity risk arising from the co-existence of both problems has been rarely investigated. Most existing work that analyzes these two problems in one model mainly focuses on *how* banking crises evolve, rather than *why* the banking industry arrives at the brink of collapse. Therefore, liquidity shortage is usually introduced as an exogenous shock, instead of a strategic outcome. However, as stated in Acharya (2009), "... Such partial equilibrium approach has a serious shortcoming from the standpoint

of understanding sources of, and addressing, inefficient systemic risk... ” In other words, if we admit that it is equally important to establish proper regulatory rules *ex ante* as it is to bailout the failing banks *ex post*, it should be equally crucial to ask what causes the failure as to tell how severe the crisis can be, i.e., systemic liquidity risk should be an endogenous phenomenon.

Recent work starts analyzing endogenous incentives for systemic risk. Acharya (2009) and Acharya & Yorulmazer (2008) define these incentives as the correlation of portfolio selection, i.e., when the return of a bank’s investment has a “systemic factor”, the failure of one bank conveys negative information about this factor, which makes the market participants worry about the health of the entire banking industry, increasing the bank’s probability to fail. The concern of such “informational spillover” induces the banks to herd *ex ante*, leading to an inefficiently high correlation in the banks’ portfolio choices. However, since illiquidity problem is not explicitly modelled in their works, liquidity regulation doesn’t play any role (in contrast to this paper).

Other recent endogenous approaches to modelling systemic liquidity risk include Wagner (2009, where inefficiency comes from the externalities of bank runs), Korinek (2008, with inefficiency coming from the fact that financial institutions don’t internalize the impact of asset prices on the production sector), etc. However, to the best of my knowledge, models addressing joint illiquidity-insolvency problem and its impact on macro policy still seem to be rare, if not absent. In this sense, this paper contributes to understanding this new feature and the lessons for banking regulation.

The mostly closely related work is probably the model of Bolton, Santos and Scheinkman (2009a, a.k.a. BSS as in the following). The feature that the market participants can hardly distinguish between illiquidity and insolvency is captured in their model, while they mainly focus on the supply side of liquidity, i.e., liquidity from financial institutions’ own cash reserve (*inside liquidity*) or from the proceeds from asset sales to the other investors with longer time preference (*outside liquidity*), and the timing perspective of liquidity trading. This paper takes BSS’s view that (outside) liquidity shortage arises from the banks’ coordination failure, but the

timing of liquidity trading is not going to be my focus. Rather, I provide a different explanation of systemic liquidity risk, i.e., liquidity under-provision may come from the banks' incentive of free-riding on each others' liquidity supply (this is not covered in BSS since they restrict attentions to pure strategy equilibria). Clear-cut results from a compact and flexible model in this paper lead to clear-cut policy implications. What's more, since financial contracts in BSS are real, they (BSS, 2009b) conclude that efficiency can be restored by central banks' credible supporting (real) asset prices. However, in practice, instead of redirecting real resources to the financial sector, central banks can only increase the supply of fiat money and support the nominal prices. If we take this into account, nominal liquidity injection from central banks may crowd out market liquidity supply without improving efficiency. Therefore policy makers should take a more careful view on designing regulatory rules and bailout policies.

1.3 Structure of the paper

SECTION 2 presents the baseline model with real deposit contracts. SECTION 2.3 presents the solution to the central planner's problem. Then SECTION 2.4 characterizes the market equilibrium and shows how it deviates from the reference point, the central planner's solution. In the following sections, the regulatory policies which have been proposed to fix the inefficiencies are carefully examined. The failure of liquidity regulation is analyzed in SECTION 3.1, and an alternative scheme with additional taxation is proposed. It is shown in SECTION 4 that equity requirements become too costly in the presence of both illiquidity and insolvency problems, therefore an improved regulatory scheme combining liquidity regulation and minimum level of equity ratio is discussed. SECTION 5 concludes.

2 The model

In this section the deposit contracts are assumed to be real, i.e., the central bank as a fiat money issuer is absent in the model. The model is similar as that from Cao

& Illing (2008); the key differences are (1) the payoff structure of the risky assets; (2) the information structure.

2.1 *The agents, time preferences, and technology*

In this economy, there are three types of agents: investors, banks (run by bank managers) and entrepreneurs. All agents are risk neutral. The economy extends over 3 periods, $t = 0, 1, 2$, and the details of timing will be explained in the next section. We assume that

- (1) There is a continuum of investors each initially (at $t = 0$) endowed with one unit of resources. The resource can be either stored (with a gross return equal to 1) or invested in the form of bank deposits;
- (2) There are a finite number N of banks actively engaged in Bertrand competition for investors' deposits. Using the deposits, the banks as financial intermediaries can fund the projects which are run by the entrepreneurs;
- (3) There is a continuum entrepreneurs of two types, denoted by type i , $i = 1, 2$. Each type of entrepreneurs is characterized by the return R_i of their projects
 - Type 1 projects (safe projects) are realized early at period $t = 1$ with a certain return $R_1 > 1$;
 - Type 2 projects (risky projects) give a higher return $R_2 > R_1 > 1$. These projects may be realized at $t = 1$, but they may also be delayed until $t = 2$ or fail with zero return.

The exact payoff structure of type 2 projects is shown in FIGURE 1.

- (1) With probability p the projects are realized in $t = 1$. For those projects with early returns
 - (a) with probability η the project is successful, returning R_2 ;
 - (b) With probability $1 - \eta$ the project fails, returning 0.
- (2) With probability $1 - p$ the project is delayed until $t = 2$. For those projects with late returns
 - (a) with probability η the project is successful, returning R_2 ;

(b) With probability $1 - \eta$ the project fails, returning 0.

The values of p and η , however are not known at $t = 0$. They will be only revealed between 0 and 1 at some intermediate period, call it $t = \frac{1}{2}$. In the following, we are interested in the case of aggregate illiquidity / insolvency shocks. We model them in the simplest way. Assume that p can take three values, $p_L < \bar{p} < p_H$, and η can take three values as well, $\eta_L < \bar{\eta} < \eta_H$. To concentrate on the cases where there is a demand for liquidity, we assume that $\eta R_2 > R_1$ such that the expected return of risky assets is higher than that for safe asset, but $p\eta R_2 < R_1$ such that the early return of risky asset is lowerer than the return for safe asset.

At $t = \frac{1}{2}$, $p \cdot \eta$, or the early return from the risky projects, becomes public information. It can take two values, $(p \cdot \eta)_H$ and $(p \cdot \eta)_L$, but no player knows the exact values of p and η . Furthermore, assume that there can be only one shock at $t = 1$, i.e., it may be either p or η that takes its “extreme” value, but not both. Assume that $(p \cdot \eta)_L = \bar{p} \cdot \eta_L = \bar{\eta} \cdot p_L < \bar{p} \cdot \eta_H = \bar{\eta} \cdot p_H = (p \cdot \eta)_H$, and $(p \cdot \eta)_H$ occurs with probability π . Therefore,

- (1) If one observes $(p \cdot \eta)_H$, it may come from either p_H (with probability σ) or η_H (with probability $1 - \sigma$);
- (2) If one observes $(p \cdot \eta)_L$, it may come from either p_L (with probability σ) or η_L (with probability $1 - \sigma$).

This setting captures the fact that both solvency and liquidity risks are relevant concerns in the banking industry. The value p defines how likely the cash flow is realized early, i.e., the liquidity of the risky projects, and η defines the quality of the projects — or, how likely the banks stay solvent.

Investors are impatient so that they want to consume early (at $t = 1$). In contrast, both entrepreneurs and bank managers are indifferent between consuming early ($t = 1$) or late ($t = 2$). To motivate the role of liquidity, we assume that resources of investors are scarce in the sense that there are more projects of each type available than the aggregate endowment of investors. Due to the hold up problem as modelled in Hart & Moore (1994), entrepreneurs can only commit to pay a fraction $p <$

Timing of the model:

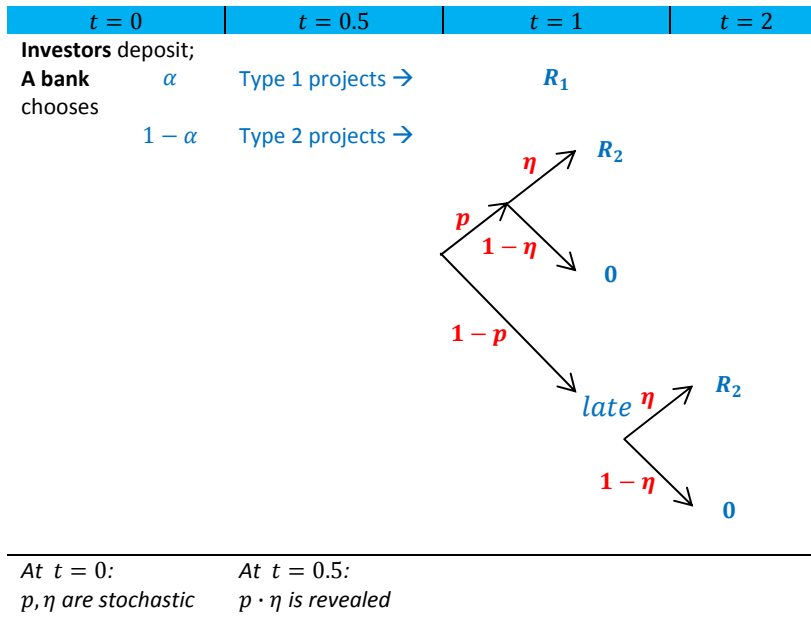


Fig. 1. The timing of the model

$\gamma < 1$ of their return. Banks' role as intermediaries is justified by the fact that they have superior collection skills (a higher γ). In a frictionless economy (in the absence of hold up problem), total surplus would go to the investors. They would simply put all their funds in early projects and capture the full return. However, the hold up problem prevents realization of such an outcome, creating a demand for liquidity. Since there is a market demand for liquidity only if investors' funds are the limiting factor, we concentrate on deviations from this market outcome. With investors' payoff as the relevant criterion, we analyze those equilibria coming closest to implement the frictionless market outcome.

Following Diamond & Rajan (2001), banks offer deposit contracts with a fixed payment d_0 payable at any time after $t = 0$ as a credible commitment device not to abuse their collection skills. The threat of a bank run disciplines bank managers to fully pay out all available resources pledged in the form of bank deposits. Deposit contracts, however, introduce a fragile structure into the economy: Whenever investors have doubts about their bank's liquidity (the ability to pay investors the promised amount d_0 at $t = 1$), they run on the bank at the intermediate date, forcing

the bank to liquidate all its projects (even those funding entrepreneurs with safe projects) at high costs: Early liquidation of projects gives only the inferior return $c < 1$. In the following, we do not consider pure sunspot bank runs of the Diamond & Dybvig type. Instead, we concentrate on the runs happening if liquid funds are not sufficient to payout investors.

Limited liability is assumed throughout the paper. All the financial contracts only have to be met with the debtors' entire assets. For the deposit contracts between investors and banks, when a bank run happens only the early withdrawers receive promised payout d_0^i ; for the liquidity contracts between banks and entrepreneurs at $t = 1$, although in equilibrium the contracted interest rate is bid up by the competing banks to the level that the entrepreneurs seize all the return from the risky projects in the good state of the world at $t = 2$ (the details will be explained later), the entrepreneurs cannot claim more than the actual yields in the bad state.

2.2 *Timing and events*

The timing and events of the model are shown in FIGURE 1. At date $t = 0$, banks competing for funds offer deposit contracts with payment d_0 which maximize expected return of investors. Banks compete by choosing the share α of deposits invested in type 1 projects, taking their competitors choice as given. Investors have rational expectations about each bank's default probability; they are able to monitor all banks' investment. At this stage, the share of type 2 projects that will be realized early is not known.

At date $t = \frac{1}{2}$, the return of type 2 projects that will be realized at $t = 1$, $p \cdot \eta$, is revealed, so does the expected return of the banks at $t = 1$. A bank would experience a run if it cannot meet the investors' demand. If this happens, all the assets — even the safe projects — have to be liquidated.

Those banks which are not run trade with early entrepreneurs in a perfectly competitive market for liquidity at $t = 1$, clearing at interest rate r . Note that because of the hold up problem, entrepreneurs retain a rent — their share $1 - \gamma$ in the projects'

return. Since early entrepreneurs are indifferent between consuming at $t = 1$ or $t = 2$, they are willing to provide liquidity (using their rent to deposit at banks at $t = 1$ at the market rate r). Banks use the liquidity provided to pay out investors. In this way, impatient investors can profit indirectly from the investment in high yielding long term projects. So banking allows the transformation between liquid claims and illiquid projects.

At date $t = 2$, the banks collect the return from the late projects and pay back the early entrepreneurs at the predetermined interest rate r .

2.3 The central planner's constrained efficient solution

If all the agents are patient, it is ex ante optimal to allocate all the resources to the high yield risky projects so that the expected aggregate return is maximized. However, because the investors are impatient and there is no way to reshuffle the output between periods, the central planner needs to take the investors' expected return as relevant criteria.

Since $p\eta R_2 < R_1$, in the absence of hold up problems, the central planner should only invest in safe projects, maximizing the output at period 1. But due to the hold-up problem caused by entrepreneurs, the central planner can implement only a constrained efficient solution: she invests a share α on the safe assets, and α depends on the type of the risk.

Proposition 2.1 *The optimal solution for the central planner's problem is:*

- (1) *In the absence of aggregate risk, the planner invests the share $\alpha = \frac{\gamma - p}{(\gamma - p) + (1 - \gamma) \frac{R_1}{\eta R_2}} = \frac{1}{1 + (1 - \gamma) \frac{R_1}{\eta R_2(\gamma - p)}}$ in liquid projects and the investors' return is maximized at $\gamma E[R] = \gamma[\alpha R_1 + (1 - \alpha)\eta R_2]$;*
- (2) *In the presence of aggregate risk, the central planner implements the following state contingent strategy, depending on the probability π for $(p \cdot \eta)_H$ being realized: The planner invests the share $\alpha_H = \frac{1}{1 + (1 - \gamma) \frac{R_1}{\gamma E[R_2|(p \cdot \eta)_H] - (p \cdot \eta)_H R_2}}$, in which $E[R_2|(p \cdot \eta)_s] = (p \cdot \eta)_s R_2 + [(1 - \bar{p})\bar{\eta} + (1 - \bar{p} - \sigma)(\eta_s - \bar{\eta})]R_2$*

($s \in \{H, L\}$), in liquid projects as long as $\bar{\pi}'_2 = \frac{\gamma E[R_L] - \kappa}{\gamma E[R_H] - \kappa + \gamma E[R_L] - \gamma E[R_{L|H}]}$, in which $\gamma E[R_s] = \gamma(\alpha_s R_1 + (1 - \alpha_s) E[R_2|(p \cdot \eta)_s])$ ($s \in \{H, L\}$), $\kappa = \alpha_H R_1 + (1 - \alpha_H)(p \cdot \eta)_L R_2$, $\gamma E[R_{L|H}] = \gamma(\alpha_L R_1 + (1 - \alpha_L) E[R_2|(p \cdot \eta)_H])$, and the share $\alpha_L = \frac{1}{1 + (1 - \gamma) \frac{R_1}{\gamma E[R_2|(p \cdot \eta)_L] - (p \cdot \eta)_L R_2}}$ otherwise, that is, for $0 \leq \pi < \bar{\pi}'_2$. \square

Proof See APPENDIX A.1. \square

When there is no aggregate risk, i.e., $p \cdot \eta$ is deterministic, the central planner implements the α that maximizes the investors' return. It can be seen that $\frac{\partial \alpha}{\partial \eta} > 0$, i.e., when insolvency risk is less severe, illiquidity problem dominates so that more funds should be invested on the safe assets. Moreover, $\frac{\partial \alpha}{\partial p} < 0$ implies that more funds should be invested on the safe assets when the long term projects get more illiquid. In the presence of aggregate risk, the central planner faces the tradeoff between reaping the high return from the risky projects in the good state (which corresponds to the lower α_H) and securing the return from the safe projects in the bad state (which corresponds to the higher α_L). The solution is hence a contingent plan which depends on the probability π .

2.4 The market equilibrium

In this section, we will characterize the market equilibrium with banks as financial intermediaries. For the simplest case, if there is no aggregate uncertainty and $p \cdot \eta$ is deterministic, the market equilibrium of the model is characterized by the bank i 's strategic profile (α_i, d_{0i}) , $\forall i \in \{1, \dots, N\}$ such that

- Bank i 's profit is maximized by

$$\alpha_i = \arg \max_{\alpha_i \in [0, 1]} \gamma \left\{ \alpha_i R_1 + (1 - \alpha_i) \left[p \eta R_2 + \frac{(1 - p) \eta R_2}{r} \right] \right\}; \quad (1)$$

- Bank i makes zero profit from offering deposit contract d_{0i}

$$d_{0i} = \max_{\alpha_i \in [0, 1]} \gamma \left\{ \alpha_i R_1 + (1 - \alpha_i) \left[p \eta R_2 + \frac{(1 - p) \eta R_2}{r} \right] \right\}; \quad (2)$$

- It is not profitable to deviate from (α_i, d_{0i}) unilaterally;

- The market interest rate
 - When the aggregate liquidity supply at $t = 1$ is equalized by the aggregate demand, $r \geq 1$;
 - When there is excess liquidity supply at $t = 1$, $r = 1$.

If there is no aggregate uncertainty the market equilibrium is in line with the solution of the social planner's problem which is constrained-efficient: Banks will invest such that — on aggregate — they are able to fulfill investors' claims in period 1, so there will be no run.

Proposition 2.2 *If there is no aggregate uncertainty the optimal allocation of the social planner's problem is the same as the allocation of market equilibrium, which is characterized by*

- All banks set $\alpha = \frac{\gamma-p}{(\gamma-p)+(1-\gamma)\frac{R_1}{\eta R_2}} = \frac{1}{1+(1-\gamma)\frac{R_1}{\eta R_2(\gamma-p)}}$;
- The market interest rate $r = 1$. \square

Proof See APPENDIX A.2. \square

The problem becomes complicated when there is aggregate uncertainty. When $(p \cdot \eta)_s$ ($s \in \{H, L\}$) is revealed in $t = \frac{1}{2}$, the expected return of the risky projects at $t = 2$ is given by

$$R_2^s = [(1 - \bar{p})\bar{\eta} + (1 - \bar{p} - \sigma)(\eta_s - \bar{\eta})]R_2, \quad (3)$$

and the aggregate expected return from the risky projects is

$$\begin{aligned} E[R_2|(p \cdot \eta)_s] &= (p \cdot \eta)_s R_2 + [(1 - \bar{p})\bar{\eta} + (1 - \bar{p} - \sigma)(\eta_s - \bar{\eta})]R_2 \\ &= [\bar{\eta}\sigma + (1 - \sigma)\eta_s]R_2. \end{aligned} \quad (4)$$

Since $\eta_H > \eta_L$, $E[R_2|(p \cdot \eta)_H] > E[R_2|(p \cdot \eta)_L]$.

If there's only illiquidity risk as in Cao & Illing (2008, 2010b), the expected return from the risky projects is just R_2 (the only thing that matters is the timing of cash flow). Now with co-existence of insolvency risk, such return is determined by the probability and scale of insolvency, as (4) suggest: In good time, the confidence

in the risky assets (more likely to have good quality) raises the expected return (hence asset price at $t = 1$), and vice versa.

The market equilibrium is then characterized in the following proposition:

Proposition 2.3 *The market equilibrium depends on the value of π , such that*

- (1) *There is a symmetric pure strategy equilibrium such that all the banks set α_H as long as $\pi > \bar{\pi}_2 = \frac{\gamma E[R_L] - c}{\gamma E[R_H] - c}$. In addition,*
 - (a) *At $t = 0$ the banks offer the investors a deposit contract with $d_0 = \gamma E[R_H]$;*
 - (b) *The banks survive at $(p \cdot \eta)_H$, but experience a run at $(p \cdot \eta)_L$;*
 - (c) *The investors' expected return is $E[R(\alpha_H, c)] = \pi d_0 + (1 - \pi)c$;*
- (2) *There is a symmetric pure strategy equilibrium such that all the banks set α_L as long as $0 \leq \pi < \bar{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma E[R_2|(p \cdot \eta)_H] - c}$. In addition,*
 - (a) *At $t = 0$ the banks offer the investors a deposit contract with $d_0 = \gamma E[R_L]$;*
 - (b) *The banks survive at both $(p \cdot \eta)_H$ and $(p \cdot \eta)_L$;*
 - (c) *The investors' expected return is $E[R(\alpha_L)] = d_0$;*
- (3) *When $\pi \in [\bar{\pi}_1, \bar{\pi}_2]$ there exists no symmetric pure strategy equilibrium. Moreover, there exists a mixed strategy equilibrium in which*
 - (a) *With probability θ one bank chooses to be a free-rider — setting $\alpha_r^* = 0$, offering high return for investors at $(p \cdot \eta)_H$ and are run at $(p \cdot \eta)_L$; and with probability $1 - \theta$ the bank chooses to be prudent — setting $\alpha_s^* > 0$ and surviving both $(p \cdot \eta)_H$ and $(p \cdot \eta)_L$;*
 - (b) *At $t = 0$ a free-riding bank offers a deposit contract with higher return $d_0^r = \gamma \left[(p \cdot \eta)_H R_2 + \frac{R_2^H}{r_H} \right]$, but the bank is run when $(p \cdot \eta)_L$ is observed; a prudent bank offers a deposit contract with lower return $d_0^s = \gamma \left[\alpha_s^* R_1 + (1 - \alpha_s^*) (p \cdot \eta)_H R_2 + \frac{(1 - \alpha_s^*) R_2^H}{r_H} \right]$, but the bank survives in both states;*
 - (c) *The expected returns for both types are equal, and the probability θ is determined by market clearing condition, which equates liquidity supply and demand in both states. \square*

Proof See APPENDIX A.3. \square

PROPOSITION 2.3 says that when π is low the banks coordinate on the higher α_L to always be prepared for the bad state, while when π is high the banks coordinate on the lower α_H to reap the high return in the good state since the risk of experiencing a bank run is rather low. But what makes the model more interesting is the equilibrium for intermediate values of π . In this case choosing α_H is not optimal since the cost of bank run is still high. But if all the banks choose α_L , there will be excess liquidity at $t = 1$ when the good state occurs. A bank anticipating this event has a strong incentive to free-ride, investing all the funds in the risky projects to reap the benefit of excess liquidity in the good state. Those prudent banks which still invest on the safe projects have to set a lower $\alpha_s^* < \alpha_L$ to cut down the opportunity cost of holding liquid assets. In the end, there will be a mixed strategy equilibrium. This echoes the finding in Allen & Gale (2004) that incomplete financial markets lead to mixed strategy equilibrium.

Moreover, at $t = \frac{1}{2}$ when a state of the world is realized, there is an uncertainty about the true type of the risk. The potential illiquidity and insolvency risks will have contradicting impacts on the prudent banks' decision of α_s^* . Suppose $(p \cdot \eta)_H$ is revealed at $t = \frac{1}{2}$:

- (1) It may imply a lower insolvency risk (higher η) at $t = 2$, therefore the value of risky assets at $t = 1$ gets higher so that the banks are able to get more liquidity from the entrepreneurs (hence, offer higher d_0^s for the investors at $t = 0$). Such “income effect” encourages prudent banks to set a higher α_s^* ;
- (2) It may imply less delay (higher p) for the risky projects, making it easier to fulfill d_0^s . Such “substitution effect” discourages prudent banks to set higher α_s^* .

The exact value α_s^* in equilibrium then depends on the cost of the banks' liquidity financing at $t = 1$, i.e., the interest rate r_H . Since r_H is bid up by the free-riders, it reflects the incentive for free-riding, which hinges on the probability of being in a good state, π :

- (1) When π is just a bit higher than $\bar{\pi}_1$, the profitability of free-riding in the good state is not much higher than being prudent. Therefore, there won't be many

- free-riders and r_H won't be that high. In this case “substitution effect” dominates and prudent banks will choose to set a higher α_s^* ;
- (2) When π is much higher than $\bar{\pi}_1$, the profitability of free-riding is much higher. Therefore, there will be many free-riders and r_H will be high. In this case “income effect” dominates and prudent banks will choose to set a lower α_s^* .

The investors' expected return in equilibrium as a function of π is summarized in FIGURE 2.

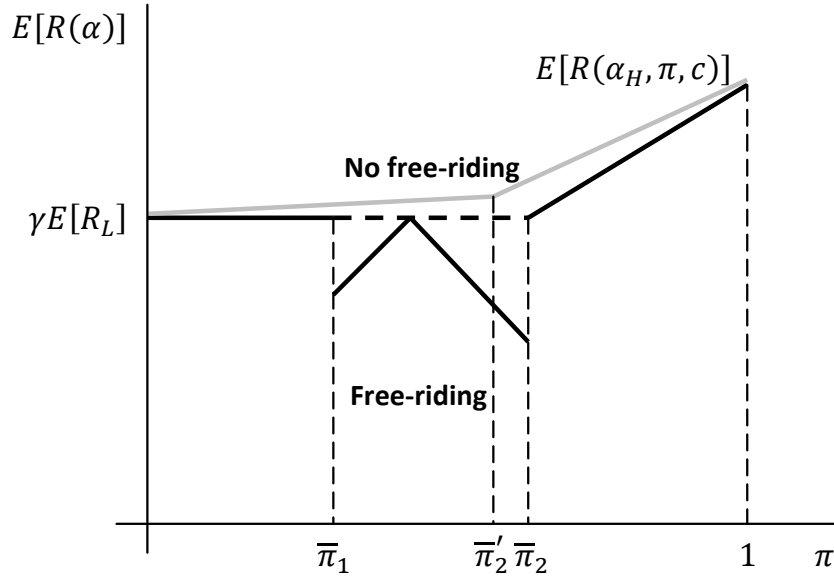


Fig. 2. Investors' expected return in the market equilibrium

Comparing with the solution of the central planner's problem, when the liquidity and insolvency problem coexist, the inefficiencies arise from: (1) the inferior mixed strategy equilibrium, and (2) the costly bank runs when π is high. Banking regulation is therefore needed to restore the efficiency. In the next section, we will examine to what extent regulatory policies can cope with these inefficiencies.

3 Liquidity regulation, nominal contract and the lender of last resort policy

One standard policy to cope with liquidity shortage is to introduce liquidity regulation: Banks are required to invest a minimum level $\underline{\alpha}$ on the safe projects, and

only those who observe the requirement will be offered the lifeboat when there's liquidity shortage. Usually such lender of last resort is the central bank, who is able to create fiat money at no cost.

In this section, we add the central bank as the fourth player into the model. The timing of the model is

- (1) At $t = 0$ the banks provide nominal deposit contract to investors, promising a fixed nominal payment d_0 at $t = 1$. The central bank announces a minimum level $\underline{\alpha}$ of investment on safe projects as the requirement for the banks' entry into the banking industry and the prerequisite for receiving liquidity injection;
- (2) At $t = \frac{1}{2}$ the banks decide whether to borrow liquidity from the central bank. If yes, the central bank will provide liquidity for the banks, provided they fulfill the requirement $\underline{\alpha}$;
- (3) At $t = 1$, the liquidity injection with the banks' illiquid assets as collateral is done so that the banks are able to honor their nominal contracts, which reduces the real value of deposits just to the amount of real resources available at that date;
- (4) At $t = 2$ the banks repay the central bank by the return from the late projects, with gross nominal interest rate $r^M \geq 1$ agreed at $t = 1$.

Since the central bank doesn't produce real goods, rather, they increase liquidity supply by printing fiat money at zero cost, therefore all financial contracts now have to be nominal, i.e., one unit of money is of equal value to one unit real good in payment and central bank's liquidity injection inflates the nominal price by *cash-in-the-market principle* à la Allen & Gale (2004) — the nominal price is equal to the ratio of amount of liquidity (the sum of money and real goods) in the market to amount of real goods. However, the welfare criterion is still based on the *real* goods received by the investors.

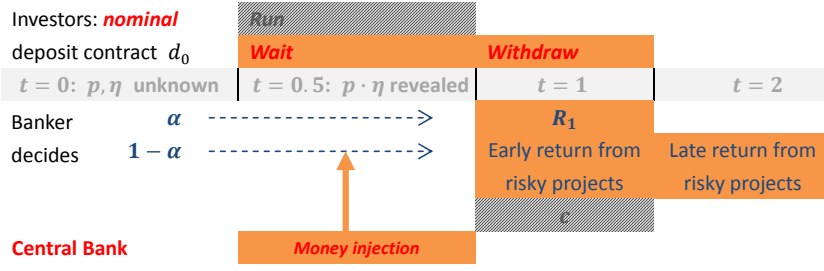


Fig. 3. The timing of the model with central bank

3.1 Liquidity regulation with conditional bailout

In the presence of nominal contracts as well as the central bank as the lender of last resort, as Cao & Illing (2010b) argues, the optimal policy is to restore the efficient allocation as that of PROPOSITION 2.1. Therefore, the liquidity requirement $\underline{\alpha} = \alpha_L$ for $0 \leq \pi \leq \bar{\pi}_2$, and $\underline{\alpha} = \alpha_H$ for $\bar{\pi}_2 < \pi \leq 1$. Moreover, the troubled banks should get liquidity injection at the lowest cost, i.e., $r^M = 1$.

With $\underline{\alpha} = \alpha_L$ as a requirement for entry, the inefficient mixed strategy equilibrium is completely eliminated and the constrained efficiency is restored for $0 \leq \pi \leq \bar{\pi}_2$. For $\bar{\pi}_2 < \pi \leq 1$, with $\underline{\alpha} = \alpha_H$ the banks can meet the deposit contract with their real return at $t = 1$ if $(p \cdot \eta)_H$ is revealed

$$d_0 = \alpha_H \gamma R_1 + (1 - \alpha_H) \gamma E [R_2 | (p \cdot \eta)_H] = d_0 |_{(p \cdot \eta)_H}.$$

If $(p \cdot \eta)_L$ is revealed, the banks need liquidity injection to meet the nominal contracts. However, since r^M is bounded by 1, the central bank can only inject liquidity up to the expected return of the risky assets. Therefore, the maximum nominal payoff the depositors can get is

$$d_0 |_{(p \cdot \eta)_L} = \alpha_H \gamma R_1 + (1 - \alpha_H) \gamma E [R_2 | (p \cdot \eta)_L] < d_0 \quad (5)$$

— the banks will still be run even if they obtain the promised lifeboat from the central bank, and the outcome is no different from that in the market equilibrium.

The scheme fails to eliminate the inefficient bank runs for $\pi > \bar{\pi}_2$.

With both illiquidity and insolvency risks, the value of the risky assets is depressed when the bad state is revealed, which makes the banks unable to get as much liquidity as they may need. Therefore, in contrast to the models with pure illiquidity risk such as Cao & Illing (2010b), pure liquidity regulation with conditional bailout is no longer sufficient to eliminate the costly bank runs.

3.2 Conditional liquidity injection with procyclical taxation

The failure of pure liquidity regulation comes from the fact that the potential insolvency risk adds an extra cost to stabilizing the financial system. This implies that the regulator needs to find a second instrument for covering such cost, for example, an additional banking tax: In addition to the scheme in SECTION 3.1, a tax has to be paid at $t = 1$ if $(p \cdot \eta)_H$ is observed, and the troubled banks will be bailed out with liquidity injection plus such the tax revenue if $(p \cdot \eta)_L$ is observed.

Such augmented scheme works as follows: At $t = 0$, a minimum liquidity requirement $\underline{\alpha}_T$ is imposed on all banks and at $t = 1$ the banks are taxed away a fixed amount $T_H \geq 0$ out of their revenue if $(p \cdot \eta)_H$ is observed. The banks are bailed out with liquidity injection plus the tax revenue if $(p \cdot \eta)_L$ is observed, and in this case the banks pay no tax, $T_L = 0$.

To find the optimal policy, first consider the high values of π . To eliminate the bank runs, T_H should be so high that the central bank has just sufficient resource to cover the gap left by liquidity injection, i.e.,

$$\begin{aligned} \underline{\alpha}_T \gamma R_1 + (1 - \underline{\alpha}_T) \gamma E [R_2 | (p \cdot \eta)_H] - T_H &= \underline{\alpha}_T R_1 + (1 - \underline{\alpha}_T) (p \cdot \eta)_H R_2 - T_H, \\ &= d_{0,T}. \end{aligned} \quad (6)$$

and

$$\underline{\alpha}_T \gamma R_1 + (1 - \underline{\alpha}_T) \gamma E [R_2 | (p \cdot \eta)_H] - T_H$$

$$= \underline{\alpha}_T \gamma R_1 + (1 - \underline{\alpha}_T) \gamma E [R_2 | (p \cdot \eta)_L] + T_H \frac{\pi}{1 - \pi}. \quad (7)$$

Equation (6) is no different from the social planner's problem for high π , therefore, the liquidity requirement $\underline{\alpha}_T = \alpha_H$ when π is high. Equation (7) says that the tax revenue should be just sufficient to fill in the gap in the liquidity bail-out,

$$T_H = (1 - \pi) \gamma (1 - \alpha_H) (E [R_2 | (p \cdot \eta)_H] - E [R_2 | (p \cdot \eta)_L]).$$

The depositors' real return in the bad state is

$$\alpha_H R_1 + (1 - \alpha_H) (p \cdot \eta)_L R_2 + T_H \frac{\pi}{1 - \pi}.$$

When π gets lower, it would be costly to stay with α_H . The regulator should switch to $\underline{\alpha}_T = \alpha_L$ when

$$\begin{aligned} \gamma E[R_L] &> \pi (\alpha_H \gamma R_1 + (1 - \alpha_H) \gamma E [R_2 | (p \cdot \eta)_H] - T_H) \\ &\quad + (1 - \pi) \left(\alpha_H R_1 + (1 - \alpha_H) (p \cdot \eta)_L R_2 + T_H \frac{\pi}{1 - \pi} \right), \\ &= \pi \gamma E[R_H] + (1 - \pi) \kappa, \\ \pi &< \frac{\gamma E[R_L] - \kappa}{\gamma E[R_H] - \kappa} = \bar{\pi}'_{2T}. \end{aligned} \quad (8)$$

The effectiveness of the scheme is summarized in the following proposition:

Proposition 3.1 *With liquidity regulation complemented by the procyclical banking tax, the bank runs are completely eliminated. Moreover,*

- (1) *For $\pi \in [0, \bar{\pi}'_{2T}]$, banks are required to invest a share of $\underline{\alpha}_T = \alpha_L$ on the safe assets, and no banking tax is necessary. The investors' expected real return is lower than the central planner's constrained efficient solution;*
- (2) *For $\pi \in (\bar{\pi}'_{2T}, 1]$, banks are required to invest a share of $\underline{\alpha}_T = \alpha_H$ on the safe assets. The banking tax T_H is charged at $t = 1$ when $(p \cdot \eta)_H$ is revealed, and the investors' expected real return is the same as the central planner's constrained efficient solution. \square*

Proof See APPENDIX A.4. \square

However, in practice such safety funds via procyclical taxation are certainly subject to implementation difficulties. The funds have to be accumulated to a sufficient amount before they are in need, i.e., when a crisis hits. Otherwise, when a crisis comes before the funds are fully established, the government must face a public deficit which can only be covered by the future taxation revenue. Usually raising public deficits implies political debates and compromises, substantially restricting the effectiveness of such scheme. In this sense, a “self-sufficient” solution such as equity holding may be superior, which is to be studied in the next section.

4 Insolvency risk and equity requirement

As seen above, with the coexistence of both illiquidity and insolvency risks, the scheme of liquidity requirement with conditional bailout only works if an additional cost is introduced. Such cost can be either “external”, for example, establishing safety funds via taxation as the past section suggested, or “internal”, for example, covering the cost with equity holdings.

4.1 Pure equity requirement

Now suppose an equity requirement is imposed to stabilize financial system in a way that all the losses will be absorbed by equity holders. Like Cao & Illing (2010b), equity is introduced à la Diamond & Rajan (2005) such that the banks issue a mixture of deposit contract and equity for the investors. Assume that the equity holders (investors) and the bank managers equally share the profit, i.e., in the good time the level of equity k is defined as the ratio of a bank’s capital to its assets

$$k = \frac{\frac{\gamma E[R_H] - d_{0,E}}{2}}{\frac{\gamma E[R_H] - d_{0,E}}{2} + d_{0,E}}, d_{0,E} = \frac{1 - k}{1 + k} \gamma E[R_H],$$

in which $d_{0,E}$ denotes the investors' return from deposits under equity requirements.

The minimum equity requirement k should make the banks just able to survive without bank runs in the bad state, i.e., all the equity is wiped out when $(p \cdot \eta)_L$ is observed,

$$\frac{1-k}{1+k} \gamma E[R_H] = \underbrace{\alpha_H R_1 + (1 - \alpha_H)(p \cdot \eta)_L R_2}_{\kappa} = d_{0,E}, \quad (9)$$

or,

$$k = \frac{\gamma E[R_H] - d_{0,E}}{\gamma E[R_H] + d_{0,E}}.$$

Since $\frac{\partial k}{\partial (p \cdot \eta)_L} < 0$ by equation (9), banks need higher equity ratio to survive in the bad state when both (or either) of the two plagues get(s) more severe, implying a higher regulatory cost.

Now the investors' real expected return is the sum of the deposit return and the dividend from equity holding

$$\begin{aligned} & \frac{1-k}{1+k} \gamma E[R_H] \pi + (1-\pi) \kappa + \frac{\gamma E[R_H] - d_{0,E}}{2} \pi \\ &= \kappa + \frac{\gamma E[R_H] - \kappa}{2} \pi. \end{aligned} \quad (10)$$

FIGURE B.2 (APPENDIX B) visualizes the results by numerical simulation. Again, as Cao & Illing (2010b) shows, holding equity is costly when π is high (i.e., less funds are available for the relatively safe, high yields risky assets, although the costly bank runs are completely eliminated). Holding equity may be superior to the mixed strategy equilibrium depending on parameter values, but is inferior to conditional liquidity injection with procyclical taxation — because taxation revenue is entirely returned to investors as bailout funds, while in the current scheme part of the profits goes to bank managers as dividends. However, concerning the implementation difficulties of imposing an extra tax, this may be a necessary cost for both investors and regulators.

4.2 Combining equity requirement with liquidity regulation

Liquidity requirements with conditional liquidity injections work best with pure illiquidity risk, but the scheme fails when there's additional insolvency risk. On the other hand, pure equity requirements are able to stabilize the system under both settings at a relatively high cost. Now the question is: Is it possible to design a regulatory scheme that combines the advantages of these two at a lower cost?

Consider the right hand side of equation (9). If banks are required to maintain the financial stability in a self-sufficient way, in all contingencies the depositors can only receive the same expected return as in the bad state. However, since there's a positive probability that the risky assets are simply illiquid, the expected future return from the risky assets can be higher, i.e., the “fair” value of the risky assets (as the right hand side of equation (5) shows) is higher. Therefore, liquidity injection from the central bank may enable the banks to pledge for bailout funds up to the fair value of their late risky assets. And the banks only need equity to cover the gap left over by liquidity injection, it'll be much less costly for the banks to carry equity.

The proposed regulatory scheme is as follows: First, all banks are required to invest $\underline{\alpha}_E = \alpha_H$ of their funds on safe assets at $t = 0$ for high π , and $\underline{\alpha}_E = \alpha_L$ for low π (the cutoff value of π is different from $\bar{\pi}_2$, and we'll compute it later); second, all the banks are required to meet a minimum equity ratio k' for high π ¹. The banks are bailed out by liquidity injection in the form of fiat money provision when the time is bad. In this case, the regulator only needs to set k' to fill in the gap after a liquidity injection when $(p \cdot \eta)_L$ is observed, i.e.,

$$\begin{aligned} \frac{1 - k'}{1 + k'} \gamma E [R_H] &= \alpha_H \gamma R_1 + (1 - \alpha_H) \gamma E [R_2 | (p \cdot \eta)_L] \\ &= \gamma E [R_{H|L}] \\ &= d'_{0,E} \end{aligned} \tag{11}$$

¹ For sufficiently low π the banks coordinate on the safe strategy, therefore there will be no bank runs and no need for liquidity injection, hence no need for equity to cover the gap in bailout funds.

in which $k' < k$ since the right hand side of (11) is higher than that of (9), and $\alpha_H R_1 + (1 - \alpha_H) E[R_2 | (p \cdot \eta)_L]$ is denoted by $E[R_{H|L}]$. The investors' deposit return is now $d'_{0,E}$. Then when $(p \cdot \eta)_H$ is observed, the investors' real expected return is $\frac{1-k'}{1+k'} \gamma E[R_H]$. However, when $(p \cdot \eta)_L$ is observed, the investors' real expected return is κ (the right hand side of (9)) and the liquidity is injected for the banks to meet the nominal deposit contract. Therefore, the investors' real expected return is the sum of the deposit return and the dividend from equity holding

$$\begin{aligned} & \frac{1-k'}{1+k'} \gamma E[R_H] \pi + (1-\pi)\kappa + \frac{\gamma E[R_H] - d'_{0,E}}{2} \pi \\ &= \pi \gamma E[R_{H|L}] + (1-\pi)\kappa + \frac{E[R_H] - E[R_{H|L}]}{2} \gamma \pi. \end{aligned} \quad (12)$$

For sufficiently low π the banks are required to hold $\underline{\alpha}_E = \alpha_L$, and the investors' expected return is $\gamma E[R_L]$. It pays off for the banks to choose α_L instead of α_H only if they get higher expected real return than (12), i.e., when

$$\gamma E[R_L] > \pi \gamma E[R_{H|L}] + (1-\pi)\kappa + \frac{E[R_H] - E[R_{H|L}]}{2} \gamma \pi. \quad (13)$$

The solution gives the cutoff value $\bar{\pi}_2''$, which can be solved from (13) when it holds with equality

$$\bar{\pi}_2'' = \frac{\gamma E[R_L] - \kappa}{\gamma \frac{E[R_H] + E[R_{H|L}]}{2} - \kappa}.$$

The effectiveness of the scheme is summarized in the following proposition:

Proposition 4.1 *With liquidity regulation complemented by equity requirements, the bank runs are completely eliminated. The investors' expected real return is higher than that under pure equity requirements, but lower than that under liquidity regulation complemented by the procyclical banking tax.*

Proof See APPENDIX A.5. \square

FIGURE B.3 (APPENDIX B) visualizes the results by numerical simulation. Such

hybrid scheme indeed effectively reduces regulatory costs in comparison to pure equity requirement, since the banks do not have to hold that much equity to stabilize the system, i.e., regulator needs two instruments to deal with two troubles.

FIGURE B.4 (APPENDIX B) compares the investors' returns under all schemes. Again, the outcome under conditional liquidity injection with procyclical taxation is superior to all the others, since all the profits that are levied as the safety tax will be entirely returned to the investors. However, when the political cost is too high to impose an extra tax and raise public deficit, combining the advantages of liquidity regulation and equity requirement is the best self-sufficient scheme.

5 Conclusion

In the existing banking literature, illiquidity and insolvency shocks are usually insulated in the sense that market participants are assumed to have perfect knowledge about the type of the shock. This paper attempts to model the fact that financial innovation makes it harder to tell whether a financial institution is illiquid or insolvent. Such ambiguity doesn't only alter the market equilibrium outcomes, but also significantly complicates the regulator's roadmaps.

It is shown that the price of illiquid assets as collateral is inflated in the good state while depressed in the bad state. This explains why the market is awash with credit in good times but the bank lending is frozen in bad times. To maintain financial stability, liquidity regulation must be complemented by equity requirements: Pure liquidity regulation deters free-riding incentives, but is not sufficient to avoid inefficient bank runs in the bad state since the collaterals are no longer considered to be good. Therefore, banks also have to hold an additional equity buffer to cover the extra cost. An alternative complement to liquidity regulation is to introduce a banking tax, which is a reserve levied from the banks' profit in the boom and used to bail out the banks in the bust. However, raising new tax generally implies higher political cost, which is not covered in this model and left for future research.

Appendix

A Proofs

A.1 Proof of PROPOSITION 2.1

In the absence of aggregate risk, given $p \cdot \eta$, the social planner maximizes the investors' return by setting α such that

$$\alpha = \arg \max_{\alpha \in [0,1]} \gamma \left\{ \alpha R_1 + (1 - \alpha) \left[p\eta R_2 + \frac{(1 - p)\eta R_2}{r} \right] \right\},$$

and the interest rate r is determined by

$$r(1 - \gamma)[\alpha R_1 + (1 - \alpha)p\eta R_2] = \gamma(1 - \alpha)(1 - p)\eta R_2 \text{ with } r \geq 1.$$

Solve to get $\alpha = \frac{\gamma - p}{(\gamma - p) + (1 - \gamma)\frac{R_1}{\eta R_2}} = \frac{1}{1 + (1 - \gamma)\frac{R_1}{\eta R_2(\gamma - p)}}$, with $r = 1$.

In the presence of aggregate risk, the social planner's optimal α may depend on π . First, solve for the α that maximizes the investors' return for each $\pi \in [0, 1]$.

The gross interest rate offered to the entrepreneurs at $t = 1$ is no less than 1, this implies that for any given α the investors' expected payoff is

$$E[R(\alpha)] = \pi \min \{ \alpha R_1 + (1 - \alpha)(p \cdot \eta)_H R_2, \gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_H]) \} \\ + (1 - \pi) \min \{ \alpha R_1 + (1 - \alpha)(p \cdot \eta)_L R_2, \gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_L]) \},$$

which is a linear function of π . Define α_H as the α that equates $\alpha R_1 + (1 - \alpha)(p \cdot \eta)_H R_2$ and $\gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_H])$, and α_L as the α that equates $\alpha R_1 + (1 - \alpha)(p \cdot \eta)_L R_2$ and $\gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_L])$, solve to get $\alpha_H = \frac{1}{1 + (1 - \gamma)\frac{R_1}{\gamma E[R_2|(p \cdot \eta)_H] - (p \cdot \eta)_H R_2}}$ and $\alpha_L = \frac{1}{1 + (1 - \gamma)\frac{R_1}{\gamma E[R_2|(p \cdot \eta)_L] - (p \cdot \eta)_L R_2}}$. Depict $E[R(\alpha_H)] = \pi \gamma E[R_H] + (1 - \pi)\kappa$ and $E[R(\alpha_L)] = \pi \gamma E[R_{L|H}] + (1 - \pi)\gamma E[R_L]$ as FIGURE A.1 shows, in which the intersection is denoted by $\bar{\pi}'_2 = \frac{\gamma E[R_L] - \kappa}{\gamma E[R_H] - \kappa + \gamma E[R_L] - \gamma E[R_{L|H}]}$.

For any $\alpha \in (\alpha_L, 1]$,

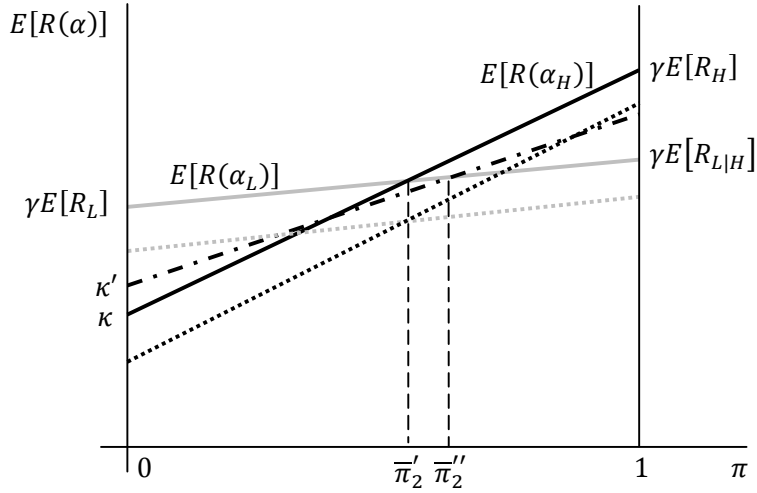


Fig. A.1. The investors' expected return for any $\alpha \in [0, 1]$. The grey line for $E[R(\alpha_L)]$, the black line for $E[R(\alpha_H)]$, the dotted grey line for those $E[R(\alpha)]$ with $\alpha \in (\alpha_L, 1]$, the dotted black line for those $E[R(\alpha)]$ with $\alpha \in [0, \alpha_H)$, and the chain line for those $E[R(\alpha)]$ with $\alpha \in (\alpha_H, \alpha_L)$.

$$E[R(\alpha)] = \pi\gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_H]) + (1 - \pi)\gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_L]) < \gamma E[R(\alpha_L)]$$

as the dotted grey lines in FIGURE A.1. For any $\alpha \in [0, \alpha_H)$, $E[R(\alpha)] = \pi[\alpha R_1 + (1 - \alpha)(p \cdot \eta)_H R_2] + (1 - \pi)[\alpha R_1 + (1 - \alpha)(p \cdot \eta)_L R_2]$. Note that $E[R(\alpha)] < \kappa$ when $\pi = 0$ and $E[R(\alpha)] < \gamma E[R_H]$ when $\pi = 1$, as the dotted black lines in FIGURE A.1.

For any $\alpha \in (\alpha_H, \alpha_L)$, $E[R(\alpha)] = \pi\gamma(\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_H]) + (1 - \pi)[\alpha R_1 + (1 - \alpha)(p \cdot \eta)_L R_2]$. Denote $\alpha R_1 + (1 - \alpha)E[R_2|(p \cdot \eta)_H]$ by $E[R_\alpha]$, and $\alpha R_1 + (1 - \alpha)(p \cdot \eta)_L R_2$ by κ' . Note that $\kappa < E[R(\alpha)] < \gamma E[R_L]$ when $\pi = 0$ and $\gamma E[R_{L|H}] < E[R(\alpha)] < \gamma E[R_H]$ when $\pi = 1$. Such $E[R(\alpha)]$ are depicted as the chain lines in FIGURE A.1.

Suppose that the intersection between $E[R(\alpha)]$ and $E[R(\alpha_L)]$ is $\bar{\pi}_2' = \frac{\gamma E[R_L] - \kappa'}{\gamma E[R_\alpha] - \kappa' + \gamma E[R_L] - \gamma E[R_{L|H}]}$. To determine the value of $\bar{\pi}_2''$, note that $\bar{\pi}_2'' \gtrless \bar{\pi}_2'$ only if

$$\frac{\gamma E[R_L] - \kappa'}{\gamma E[R_\alpha] - \kappa' + \gamma E[R_L] - \gamma E[R_{L|H}]} \gtrless \frac{\gamma E[R_L] - \kappa}{\gamma E[R_H] - \kappa + \gamma E[R_L] - \gamma E[R_{L|H}]}$$

This is equivalent to

$$\begin{aligned} & \gamma E[R_L] (\gamma E[R_H] - \gamma E[R_\alpha]) + (\gamma E[R_\alpha] - \gamma E[R_L]) \kappa + (\gamma E[R_L] - \gamma E[R_H]) \kappa' \\ & + (\gamma E[R_L] - \gamma E[R_{L|H}]) (\kappa - \kappa') \geq 0. \end{aligned} \quad (\text{A.1})$$

Using the fact that $\gamma E[R_s] = \alpha_s R_1 + (1 - \alpha_s)(p \cdot \eta)_s R_2$ ($s \in \{H, L\}$), and replace α by the linear combination of α_H and α_L , $\alpha = \omega \alpha_H + (1 - \omega) \alpha_L$ with $\omega \in (0, 1)$, the sum of the first three terms in left hand side of inequality (A.1) turns out to be

$$\gamma E[R_L] (\gamma E[R_H] - \gamma E[R_\alpha]) + (\gamma E[R_\alpha] - \gamma E[R_L]) \kappa + (\gamma E[R_L] - \gamma E[R_H]) \kappa' = 0.$$

The last term in left hand side of inequality (A.1)

$$(\gamma E[R_L] - \gamma E[R_{L|H}]) (\kappa - \kappa') > 0,$$

which implies that $\bar{\pi}_2'' > \bar{\pi}_2'$.

Combining all the cases, FIGURE A.1 shows the investors' expected return for any $\alpha \in [0, 1]$. The social planner's optimal solution is given by the frontier of the investors' expected return, which is a state contingent strategy depending on the probability π : The planner invests the share α_H in liquid projects as long as $\bar{\pi}_2' \leq \pi \leq 1$, and the share α_L in liquid projects as long as $0 \leq \pi < \bar{\pi}_2'$. \square

A.2 Proof of PROPOSITION 2.2

To show that the optimal allocation is indeed market equilibrium, one has to show that it is not profitable for any bank to deviate unilaterally. Suppose that bank i deviates by setting

(1) $\alpha_i < \alpha$. By market clearing condition, the interest rate r' is determined by

$$\begin{aligned} & r' \{ (1 - \gamma) [\alpha_i R_1 + (1 - \alpha_i) p \eta R_2] + (N - 1)(1 - \gamma) [\alpha R_1 + (1 - \alpha) p \eta R_2] \} \\ & = \gamma (1 - \alpha_i) (1 - p) \eta R_2 + (N - 1) \gamma (1 - \alpha) (1 - p) \eta R_2. \end{aligned}$$

Therefore $r' > 1$. For the non-deviators, the return for their depositors is

$$\gamma \left\{ \alpha R_1 + (1 - \alpha) \left[p \eta R_2 + \frac{(1 - p) \eta R_2}{r'} \right] \right\}$$

$$< \gamma \left\{ \alpha R_1 + (1 - \alpha) \left[p\eta R_2 + \frac{(1 - p)\eta R_2}{r} \right] \right\} = d_0 \text{ with } r = 1.$$

This means that they cannot meet the deposit contracts and the depositors will choose the deviator at $t = 0$. The deviator is able to offer at maximum

$$d'_0 = \alpha_i R_1 + (1 - \alpha_i) p\eta R_2 < \alpha R_1 + (1 - \alpha) p\eta R_2 = d_0,$$

which implies that the deviator gets worse off;

- (2) $\alpha_i > \alpha$. The rent seized by the deviator's early entrepreneurs exceeds the deviator's late return, i.e.,

$$(1 - \gamma)[\alpha R_1 + (1 - \alpha) p\eta R_2] > \gamma(1 - \alpha_i)(1 - p)\eta R_2.$$

Therefore there will be excess aggregate liquidity supply at $t = 1$ and the interest rate for the liquidity market will remain to be 1. The deposit contract that the deviator is able to offer is at maximum

$$d'_0 = \gamma[\alpha_i R_1 + (1 - \alpha_i)\eta R_2] < \gamma[\alpha R_1 + (1 - \alpha)\eta R_2] = d_0,$$

which means the deviator cannot get any depositor at $t = 0$. It is not a profitable deviation. \square

A.3 Proof of PROPOSITION 2.3

When the banks coordinate on choosing α_H , they survive when $(p \cdot \eta)_H$ is revealed but experience bank runs when $(p \cdot \eta)_L$. The investors' expected return is $\pi\gamma E[R_H] + (1 - \pi)c$. When the banks coordinate on choosing α_L , they survive in both states and the investors' expected return is $\gamma E[R_L]$. The investors' expected return is higher under α_H only if $\pi > \bar{\pi}_2 = \frac{\gamma E[R_L] - c}{\gamma E[R_H] - c}$.

When $\pi > \bar{\pi}_2$, α_H is the symmetric pure strategy equilibrium because a bank cannot profit from unilateral deviation:

- (1) If the deviator chooses α_L , its investors' expected return is lower than that of its competitors;

- (2) If the deviator chooses $\alpha > \alpha_L$, its investors' expected return is $\gamma[\alpha R_1 + (1 - \alpha)\eta R_2]$ which is decreasing in α . Therefore, such strategy α will be outbid by α_L , hence by α_H ;
- (3) If the deviator chooses $\alpha_H < \alpha < \alpha_L$, it experiences a run when $(p \cdot \eta)_L$ is revealed (which is the same for banks with α_H) and when $(p \cdot \eta)_H$ is revealed its investors' expected return is $\gamma[\alpha R_1 + (1 - \alpha)\eta R_2]$ decreasing in α so that the expected return is lower than banks with α_H ;
- (4) If the deviator chooses $\alpha < \alpha_H$, it will experience bank runs in both states, which makes such α an inferior strategy.

Following similar approach, one can show that α_L is the symmetric pure strategy equilibrium for low π , i.e., $0 \leq \pi < \bar{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma E[R_2|(p \cdot \eta)_H] - c}$. For $\pi < \bar{\pi}_2$, if the banks coordinate on α_L , there will be excess liquidity supply when $(p \cdot \eta)_H$ is revealed. A deviator may profit from the excess liquidity supply in the good state by investing all its funds in the illiquid assets, earning a return of $\gamma E[R_2|(p \cdot \eta)_H]$ when $(p \cdot \eta)_H$ is revealed. Although such deviator suffers from bank run in the bad state, the deviation can be profitable if the likelihood of having a bad state is not too high, i.e., when $\bar{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma E[R_2|(p \cdot \eta)_H] - c} \leq \pi \leq \bar{\pi}_2$.

It is also straightforward to see there doesn't exist any asymmetric pure strategy equilibrium: Given that market interest rate r is identical for all the banks, the expected return for one bank is linear in its α so that banks with lower expected returns cannot get any deposit at $t = 0$. Therefore, there is no pure strategy equilibrium for $\bar{\pi}_1 \leq \pi \leq \bar{\pi}_2$.

For $\bar{\pi}_1 \leq \pi \leq \bar{\pi}_2$ there is only equilibrium of mixed strategies as proved by Cao & Illing (2008), which is featured by

- (1) With probability θ one bank chooses to be a free-rider — setting $\alpha_r^* = 0$, offering high return for investors at $(p \cdot \eta)_H$ and are run at $(p \cdot \eta)_L$; and with probability $1 - \theta$ the bank chooses to be prudent — setting $\alpha_s^* > 0$ and surviving both $(p \cdot \eta)_H$ and $(p \cdot \eta)_L$;
- (2) At $t = 0$ a free-riding bank offers a deposit contract with higher return $d_0^r = \gamma \left[(p \cdot \eta)_H R_2 + \frac{R_2^H}{r_H} \right]$, but the bank experiences a run when $(p \cdot \eta)_L$ is observed;

a prudent bank offers a deposit contract with lower return

$$d_0^s = \gamma \left[\alpha_s^* R_1 + (1 - \alpha_s^*) (p \cdot \eta)_H R_2 + \frac{(1 - \alpha_s^*) R_2^H}{r_H} \right],$$

but the bank survives in both states;

- (3) The expected returns for both types are equal, $d_0^s = \pi d_0^r + (1 - \pi)c$, and the probability θ is determined by market clearing condition, which equates liquidity supply and demand in both states:

(a) At $(p \cdot \eta)_H$, $\theta D_r + (1 - \theta) D_s = \theta S_r + (1 - \theta) S_s$, in which

- Liquidity demand from a free-riding bank

$$D_r = d_0^r - \gamma (p \cdot \eta)_H R_2;$$

- Liquidity demand from a prudent bank

$$D_s = d_0^s - \gamma [\alpha_s^* R_1 + (1 - \alpha_s^*) (p \cdot \eta)_H R_2];$$

- Liquidity supply from the entrepreneurs of a free-riding bank

$$S_r = (1 - \gamma) (p \cdot \eta)_H R_2;$$

- Liquidity supply from the entrepreneurs of a prudent bank

$$S_s = (1 - \gamma) [\alpha_s^* R_1 + (1 - \alpha_s^*) (p \cdot \eta)_H R_2];$$

(b) At $(p \cdot \eta)_L$, only the prudent banks survive so that

$$r_L (1 - \gamma) [\alpha_s^* R_1 + (1 - \alpha_s^*) (p \cdot \eta)_L R_2] = \gamma (1 - \alpha_s^*) R_2^L. \quad \square$$

A.4 Proof of PROPOSITION 3.1

For $\pi \in (\bar{\pi}'_{2T}, 1]$, equation (7) implies that the depositors' expected return is the same in both states, therefore, bank runs are completely eliminated. Since $\kappa > c$, $\bar{\pi}'_{2T} < \bar{\pi}_2$, which means $[0, \bar{\pi}'_{2T}]$ is a subset of $[0, \bar{\pi}_2]$ where α_L maximizes the depositors' expected return in the market equilibrium.

Equation (8) is exactly the same as $E[R(\alpha_H)]$ in FIGURE A.1, implying that the investors' expected real return is the same as the central planner's constrained efficient solution for $\pi \in (\bar{\pi}'_{2T}, 1]$. \square

A.5 Proof of PROPOSITION 4.1

Equation (11) implies that the investors get the same nominal deposits returns in both states, therefore, there will be no bank runs. The investors' expected real return $\pi\gamma\frac{E[R_H]+E[R_{H|L}]}{2}+(1-\pi)\kappa$, linear in π , becomes κ when $\pi = 0$ and $\gamma\frac{E[R_H]+E[R_{H|L}]}{2}$ when $\pi = 1$. Since $\gamma\frac{E[R_H]+E[R_{H|L}]}{2} < \gamma E[R_H]$, such real return is below $E[R(\alpha_H)]$ (see APPENDIX A.1) for all $\pi \in (0, 1]$.

Compare the investors' expected real return here with that under pure equity requirements, as in equation (10). It is easily seen that equation (10) becomes κ when $\pi = 0$ and $\frac{\gamma E[R_H]+\kappa}{2}$ when $\pi = 1$. Since $\kappa < \gamma E[R_{H|L}]$, the investors' expected real return under pure equity requirements is lower for all $\pi \in (0, 1]$. \square

B Results of numerical simulations

The following figures present numerical simulations for various regulatory schemes.

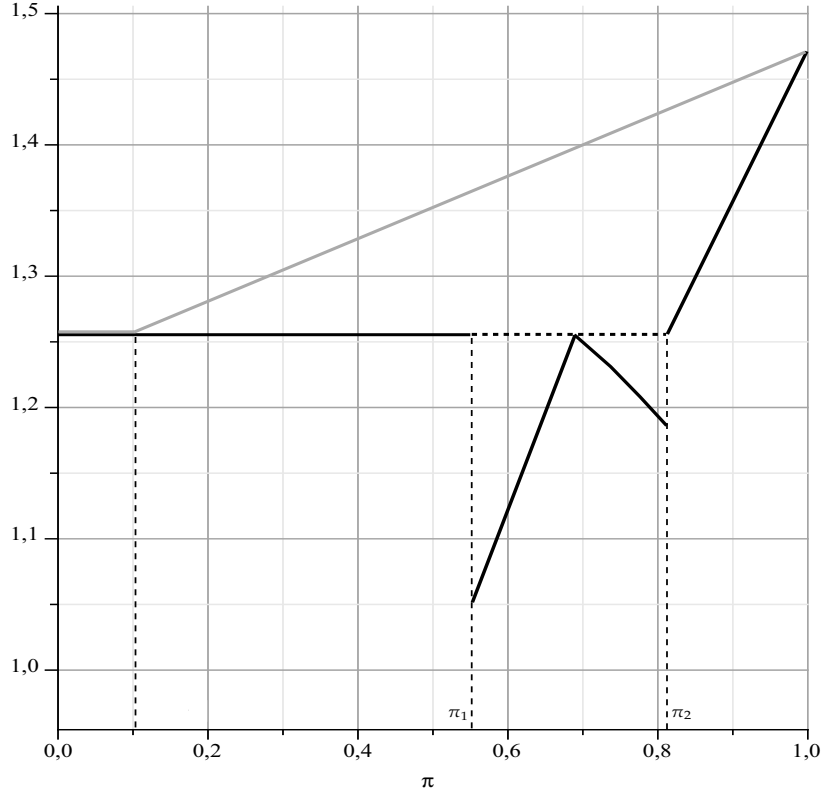


Fig. B.1. Investors' expected return in equilibrium: market economy (solid blue line) versus economy with conditional liquidity injection & procyclical taxation (solid green line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\bar{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\bar{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$.

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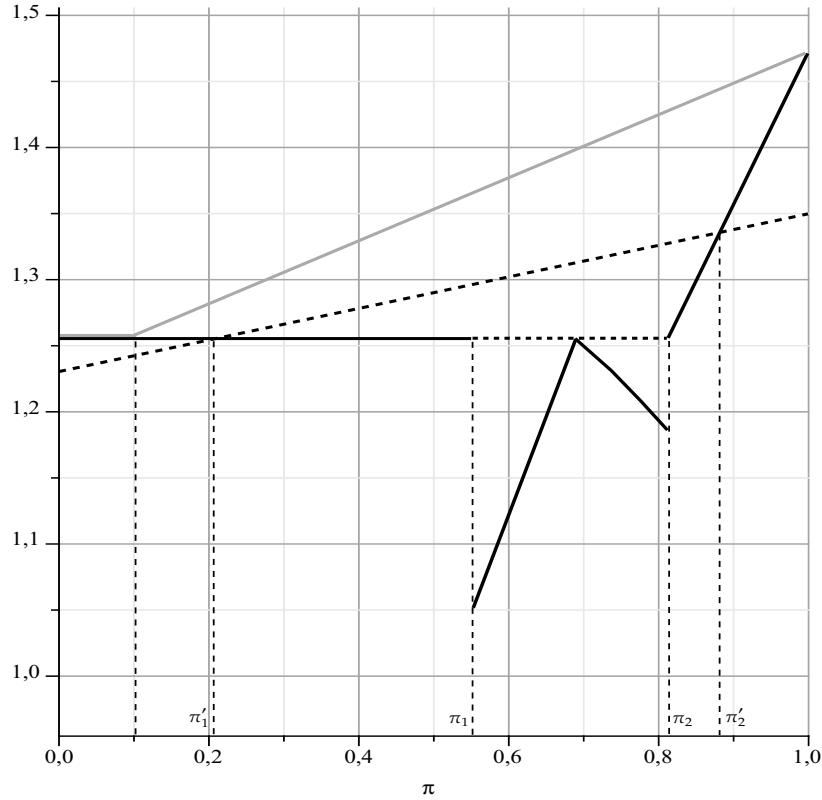


Fig. B.2. Investors' expected return in equilibrium: market economy (solid blue line) versus economy with (1) equity requirement (solid red line) (2) conditional liquidity injection & procyclical taxation (solid green line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\bar{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\bar{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$, $\zeta = 0.5$. The outcome under equity requirement is superior to that of market economy for $\pi \in [\bar{\pi}_1', \bar{\pi}_2']$.

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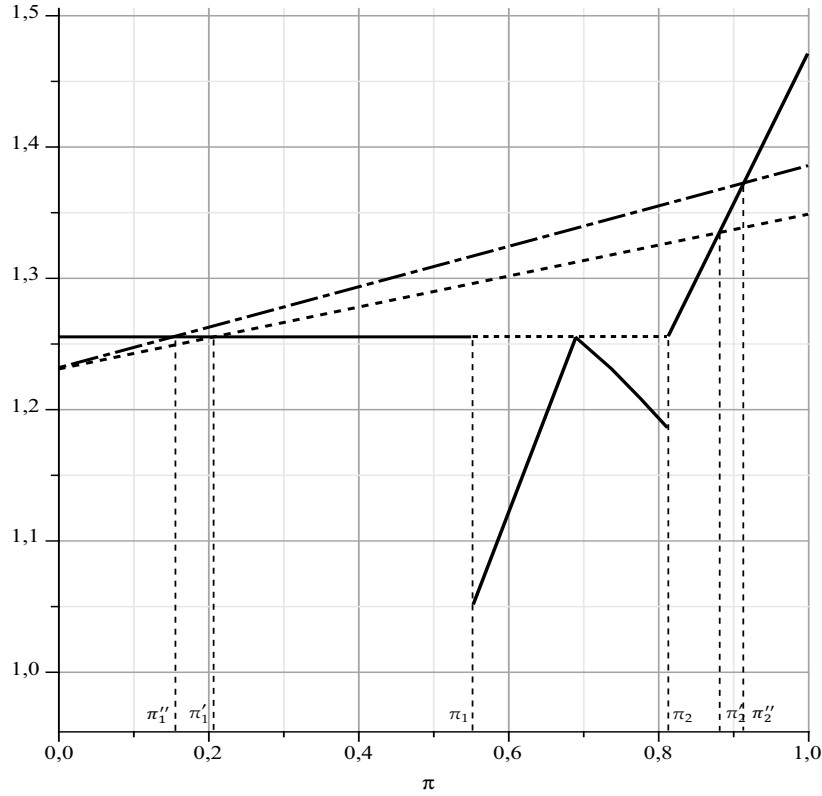


Fig. B.3. Investors' expected return in equilibrium: market economy (solid blue line) versus economy with (1) pure equity requirement (solid red line) (2) equity requirement & liquidity regulation (solid orange line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\bar{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\bar{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$, $\zeta = 0.5$. The outcome under equity requirement & liquidity regulation is superior to that of market economy for $\pi \in [\bar{\pi}_1', \bar{\pi}_2']$.

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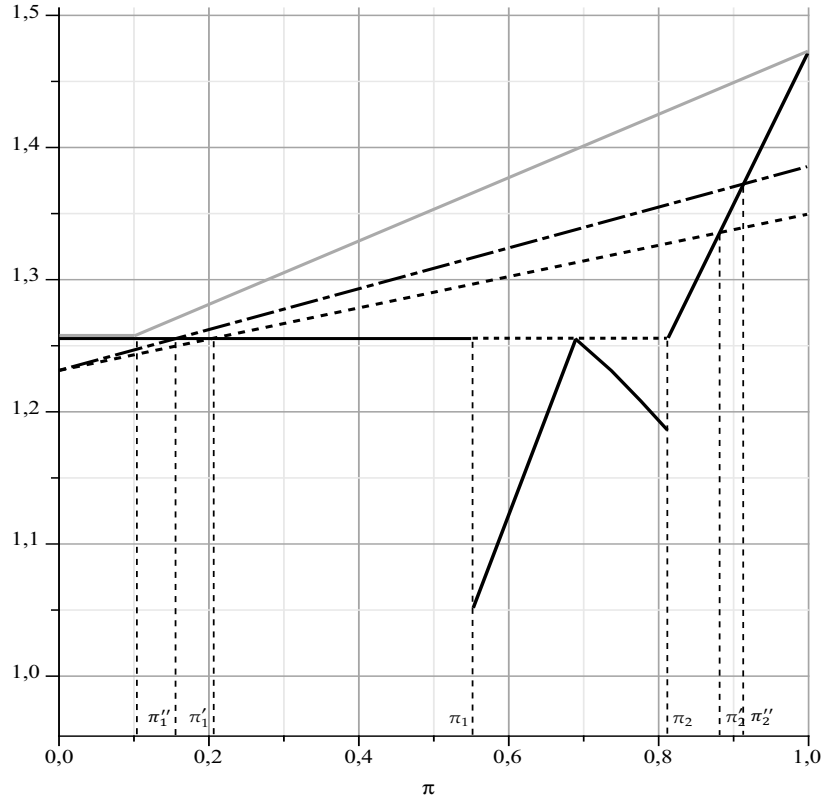


Fig. B.4. Investors' expected return in equilibrium: market economy (solid blue line) versus economy with (1) conditional liquidity injection & procyclical taxation (solid green line) (2) pure equity requirement (solid red line) (3) equity requirement & liquidity regulation (solid orange line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\bar{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\bar{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$, $\zeta = 0.5$.

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