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From Real Business Cycle to New Keynesian Economics (I): Nominal and Real Rigidities

If I were founding a university I would begin with a smoking room; next a dormitory; and then a decent reading room and a library. After that, if I still had more money that I couldn't use, I would get some text books.

—Stephen Leacock

1 Introduction

This and the next chapter present a dynamic stochastic general equilibrium based new Keynesian monetary model, which is an extension of the seminal Calvo-Yun model (Yun, 1996) but much simplified in the constrast of Schmitt-Grohé and Uribe (2005).

In the past chapters we have already seen that in an economy without frictions the monetary shocks would have little real impacts. Even for the models in which agents do value their money holdings, such as money-in-the-utility or cash-in-advance models, the goods and factor prices would be adjusted immediately in the response to monetary shocks and the real allocations would be seldom affected even in the shortest run.

However, this severely contradicts to what people observe in the reality — Monetary shocks do have real effects, and often the effects are fairly persistent. For example, as documented in the notable research by Christiano, Eichenbaum and Evans (2005, CEE in the following), shown by the solid lines with + in Figure 1, after an expansionary monetary policy shock (an unexpected fall in the nominal interest rate in the third period) one can usually observe a

- hump-shaped response of output, consumption and investment, with the peak effect occurring after about 1.5 years and returning to their pre-shock levels after about 3 years;
- hump-shaped response in inflation, with a peak response after about 2 years,
- fall in the interest rate for roughly 1 year;
- rise in profits, real wages and labor productivity; and
- an immediate rise in the growth rate of money.

Therefore in order to explain such increasing evidences that monetary policies do have effects on real output that persist for considerable periods of time, we should depart from the frictionless models and introduce some barriers to the optimal adjustments.

In Section 2 we will analyse the decision problems of the agents in a stylized economy. The agents' incentives are distorted due to the monopolistic competition à la Dixit and Stiglitz (1977), which is adopted in macro studies by Blanchard and Kiyotaki (1987). However, such distortion doesn't mean that monetary shocks have real effects, because given that the prices are flexible the agents can always respond to the shocks immediately by adjusting the nominal prices and maintain the original level of real output. Therefore, nominal rigidity in price adjustments is attached to the assumption of monopolistic competition and money is no longer neutral. And in order to magnify the persistence in the economy, real ridigity is also added into the model in the form of investment cost.

In Section 3 we will have a break in the progress to develop some insights behind the assumptions and discuss how they affect the outcome of the model. Especially we will see (1) how monopolistic competition distorts the economy and how aggregate demand externalities emerge; (2) how people may build up nominal and real rigidities in this economy.

General equilibrium and the numerical exercises are left for the next chapter.

2 The Economy

Before considering the optimal decision problems of the agents, we present a large picture of the economy and briefly sketch what is going on between the sectors.

2.1 A Brief Overview

The economy consists of three types of agents as following:

- *Households*. They are owners of the capital, and suppliers of labor forces. In each period the production takes place before the goods market opens, such that the households rent their capital stock from the past period to the firms, and provide labor to earn wage income;
- *Firms*. There are three types of firms along the value chain:
  - *Wholesale firms*. In the beginning of each period, they get capital and labor from the households as inputs, and produce differentiated intermediate products. Then they sell these products to the downstream firms — the final goods producers;
  - The intermediate goods cannot be consumed, nor stored as new capital stock, before they are assembled into the final products by the *final goods producers*. The final goods producers buy the intermediate goods from the wholesale firms as inputs, and produce the final goods as outputs;
  - Then the goods market opens. The final goods are sold in this market, they are either bought by the households as consumption, or by the *capital producers*. The capital producers buy the final goods as one input (called *investments*), and rent the capital used by the wholesale firms as the other input. The output is the new capital, and the capital market opens after the new capital is produced. The households buy the capital to adjust their capital stocks.
- *Government*. Government is the player who implement fiscal and monetary policies, following some certain *rules*.

Now let's have a look into the details.

2.2 The Households

In brief the household's problem is just like that in a baseline real business cycle model, plus the money holdings in the utility function. In each period t the household's instantaneous utility function takes the form of

$$u_t = \frac{C_t^{1-\gamma}}{1-\gamma} + \frac{a_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m} - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n}$$

in which the function is CRRA, and the agent gains utility from consumption goods C_t, real money holding $\frac{M_t}{P_t}$, and disutility from providing labor N_t. These three sources of (dis-)utility are weighted by 1, a_m and a_n respectively. There is no growth in population, so we won't bother to rewrite everything in per capita form.

In real terms, the household's resource constraint in each period t is

$$\begin{aligned}
&\frac{W_t N_t}{P_t} + Z_t K_{t-1} + \Pi_t + TR_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + Q_t(1-\delta)K_{t-1} \\
=& C_t + \frac{M_t}{P_t} + \frac{B_t}{R_t^n P_t} + Q_t K_t.
\end{aligned}$$

The left hand side of the flow budget constraint shows what the representative household gets in the beginning of this period:

- The production is implemented in the beginning of each period with the capital stock from the last period, K_{t-1}, and the household's labor supply of this period, N_t, as inputs. The household gets paid from its labor supply at the wage rate W_t, and collects the rent from renting its assets to the firms at the real rental rate Z_t. The household also holds a share of the firms (remember that the household is *representative*), therefore it gets the firms' profit Π_t. And the depreciated capital is worth $Q_t(1-\delta)K_{t-1}$, in which Q_t is the real price for the installed capital in period t;
- The household also holds some values from money and bonds, which it brings from the last period. These are evaluated by the current price level, $\frac{M_{t-1}}{P_t}$ and $\frac{B_{t-1}}{P_t}$;
- The household also obtains a transfer from the government, TR_t.

The right hand side of the flow budget constraint shows what the representative household spends during this period, after it has collected all the possible resources:

- The consumption C_t;
- The money holding to be carried over into the next period, $\frac{M_t}{P_t}$;
- The bonds holding to be carried over into the next period, $\frac{B_t}{R_t^n P_t}$. Note that the bonds get a gross nominal return R_t^n when they are carried over into the next period. $R_t^n = 1 + r_t^n$ with r_t^n being the nominal interest rate;
- The capital stock to be carried over into the next period, evaluated at the current replacement cost, $Q_t K_t$.

Since the flow budget constraint is expressed as a decentralized decision instead of the central planner's allocation, the production function with productivity shocks doesn't explicity show up here. However, since the production function (partially) pins down the wage rate and the rental rate, furtherly consumption and capital adjustment decisions and so on, all the variables in the model are in fact stochastic rather than deterministic.

Therefore the representative agent's problem can be written as

$$
\begin{aligned}
\max_{\left\{C_t, N_t, \frac{M_t}{P_t}, \frac{B_t}{P_t}, K_t\right\}_{t=0}^{+\infty}} \quad & E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \frac{a_m}{1-\gamma_m} \left(\frac{M_t}{P_t} \right)^{1-\gamma_m} - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n} \right] \right\}, \\
s.t. \quad & \frac{W_t N_t}{P_t} + Z_t K_{t-1} + \Pi_t + TR_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + Q_t (1-\delta) K_{t-1} \\
& = C_t + \frac{M_t}{P_t} + \frac{B_t}{R_t^n P_t} + Q_t K_t.
\end{aligned}
$$

Set up the Lagrangian for this problem

$$
\begin{aligned}
\mathscr{L} = E_0 \Bigg(\sum_{t=0}^{+\infty} \Bigg\{ & \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} + \frac{a_m}{1-\gamma_m} \left(\frac{M_t}{P_t} \right)^{1-\gamma_m} - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n} \right] \\
& + \lambda_t \Bigg[\frac{W_t N_t}{P_t} + Z_t K_{t-1} + \Pi_t + TR_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + Q_t (1-\delta) K_{t-1} \\
& - C_t - \frac{M_t}{P_t} - \frac{B_t}{R_t^n P_t} - Q_t K_t \Bigg] \Bigg\} \Bigg).
\end{aligned}
$$

$\forall t$, the first order conditions are

$$
\frac{\partial \mathscr{L}}{\partial C_t} = \beta^t C_t^{-\gamma} - \lambda_t = 0, \tag{1}
$$

$$
\frac{\partial \mathscr{L}}{\partial N_t} = -\beta^t a_n N_t^{\gamma_n} + \frac{\lambda_t W_t}{P_t} = 0, \tag{2}
$$

$$
\frac{\partial \mathscr{L}}{\partial \left(\frac{M_t}{P_t} \right)} = \beta^t a_m \left(\frac{M_t}{P_t} \right)^{-\gamma_m} + E_t \left(\lambda_{t+1} \frac{P_t}{P_{t+1}} \right) - \lambda_t = 0, \tag{3}
$$

$$
\frac{\partial \mathscr{L}}{\partial \left(\frac{B_t}{P_t} \right)} = E_t \left(\lambda_{t+1} \frac{P_t}{P_{t+1}} \right) - \frac{\lambda_t}{R_t^n} = 0, \tag{4}
$$

$$
\frac{\partial \mathscr{L}_t}{\partial K_t} = E_t \left[\lambda_{t+1} Z_{t+1} + \lambda_{t+1} Q_{t+1} (1-\delta) \right] - \lambda_t Q_t = 0. \tag{5}
$$

From (1) rearrange to get

$$
\lambda_t = \beta^t C_t^{-\gamma}, \tag{6}
$$

$$
\lambda_{t+1} = \beta^{t+1} E_t \left(C_{t+1}^{-\gamma} \right) \tag{7}
$$

in which (7) is just one period update of (6).

Insert (6) and (7) into (2) – (5)

$$
\frac{W_t}{P_t} C_t^{-\gamma} = a_n N_t^{\gamma_n}, \tag{8}
$$

$$C_t^{-\gamma} = a_m \left(\frac{M_t}{P_t}\right)^{-\gamma_m} + E_t \left(\beta C_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}}\right), \tag{9}$$

$$C_t^{-\gamma} = E_t \left(R_t^n \beta C_{t+1}^{-\gamma} \frac{P_t}{P_{t+1}}\right), \tag{10}$$

$$C_t^{-\gamma} = E_t \left[\frac{Z_{t+1} + Q_{t+1}(1-\delta)}{Q_t} \beta C_{t+1}^{-\gamma}\right]. \tag{11}$$

Since $\frac{P_t}{P_{t+1}} = \frac{1}{1+\pi_{t+1}}$, so equation (10) can be written as

$$C_t^{-\gamma} = E_t \left(R_t^n \beta C_{t+1}^{-\gamma} \frac{1}{1+\pi_{t+1}}\right),$$

and by Fisher's equation $R_t^n \frac{1}{1+\pi_{t+1}}$ is just the real interest return, denoted by R_t ($R_t = 1 + r_t$)

$$R_t = R_t^n \frac{1}{1+\pi_{t+1}}.$$

Therefore equation (10) can be simplified as

$$C_t^{-\gamma} = E_t \left(R_t \beta C_{t+1}^{-\gamma}\right). \tag{12}$$

(8) can be rewritten as

$$\frac{W_t}{P_t} = a_n N_t^{\gamma_n} C_t^{\gamma}. \tag{13}$$

Insert (10) into (9)

$$C_t^{-\gamma} = a_m \left(\frac{M_t}{P_t}\right)^{-\gamma_m} + \frac{C_t^{-\gamma}}{R_t^n}$$

$$\left(\frac{M_t}{P_t}\right)^{-\gamma_m} = \frac{1}{a_m}\left(1 - \frac{1}{R_t^n}\right) C_t^{-\gamma},$$

rearrange to get

$$\frac{M_t}{P_t} = \left(\frac{1}{a_m}\right)^{-\frac{1}{\gamma_m}} \left(1 - \frac{1}{R_t^n}\right)^{-\frac{1}{\gamma_m}} C_t^{\frac{\gamma}{\gamma_m}}. \tag{14}$$

And rearrange (12) and (11) to get

$$1 = E_t\left[R_t\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right], \tag{15}$$

$$1 = E_t\left[\frac{Z_{t+1} + Q_{t+1}(1-\delta)}{Q_t}\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]. \tag{16}$$

2.3 The Firms

2.3.1 Final Goods Producers

There are a number of *competitive* final goods producers in this economy, producing a homogenous final good Y_t using intermediate goods $Y_t(z)$ as inputs. z is the index of the continuum of intermediate goods whose measure is normalized to 1. The production function for the final goods is

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}}$$

in which $\epsilon > 1$. Note that this production function exhibits constant returns to scale, diminishing marginal product, and constant elasticity of substitution — the elasticity of substitution is just ϵ.

Then a representative firm's problem is to

$$\max_{Y_t(z)} \quad P_t Y_t - \int_0^1 P_t(z) Y_t(z) dz \tag{17}$$

in which $P_t(z)$ is the price of intermediate good z, asked by the upstream firms. Since the final goods producers are fully competitive and price takers, $P_t(z)$ is exogenous for them.

But as the production function is constant return to scale, the size of a firm doesn't matter. Therefore Y_t in the representative firm's profit maximization problem (PMP) is indeterminate and the question is not well defined. Note that the dual problem of PMP is the expenditure minimization problem (EMP)

$$\min_{Y_t(z)} \int_0^1 P_t(z) Y_t(z) dz,$$

$$s.t. \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}} \geq Y_t$$

in which the firm has to minimize its production cost, with some threshold output level Y_t as given.

To solve this problem, set up Lagrangian

$$\mathscr{L} = \int_0^1 P_t(z)Y_t(z)dz + \lambda\left\{\left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}} - Y_t\right\}$$

and obtain its first order condition

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial Y_t(z)} &= P_t(z) + \lambda\frac{\epsilon}{\epsilon-1}\left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon-1}{\epsilon} Y_t(z)^{\frac{\epsilon-1}{\epsilon}-1} \\
&= P_t(z) + \lambda\left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz\right]^{\frac{1}{\epsilon-1}} Y_t(z)^{-\frac{1}{\epsilon}} \\
&= P_t(z) + \lambda Y_t^{\frac{1}{\epsilon}} Y_t(z)^{-\frac{1}{\epsilon}} \\
&= 0.
\end{aligned}$$

Next we have to eliminate λ from the first order condition. Notice that the condition holds for any intermediate good, i.e. $\forall i, j \in [0,1]$ with $i \neq j$ we have

$$\begin{aligned}
P_t(i) + \lambda Y_t^{\frac{1}{\epsilon}} Y_t(i)^{-\frac{1}{\epsilon}} &= 0, \\
P_t(j) + \lambda Y_t^{\frac{1}{\epsilon}} Y_t(j)^{-\frac{1}{\epsilon}} &= 0.
\end{aligned}$$

Eliminating λ by these two equations to solve for $Y_t(i)$

$$\begin{aligned}
\frac{P_t(i)}{P_t(j)} &= \left[\frac{Y_t(i)}{Y_t(j)}\right]^{-\frac{1}{\epsilon}}, \\
Y_t(i) &= \left[\frac{P_t(j)}{P_t(i)}\right]^{\epsilon} Y_t(j).
\end{aligned}$$

The last equation says that if we define any good j as a *reference good*, then the demand for any other good $i \in [0,1]\backslash\{j\}$ can be represented via the demand for good j adjusted by the elasticity form of the relative price. Since at optimum the inequality constraint must be binding, therefore replace all the goods with the reference good and get

$$
\begin{aligned}
\left(\int_0^1 \left\{\left[\frac{P_t(j)}{P_t(i)}\right]^{\epsilon} Y_t(j)\right\}^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} &= Y_t, \\
P_t(j)^{\epsilon} Y_t(j) \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} &= Y_t.
\end{aligned}
$$

Since j is arbitrarily taken from $[0, 1]$, we can replace it with z,

$$
P_t(z)^{\epsilon} Y_t(z) \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} = Y_t. \tag{18}
$$

On the other hand, since the sector for final goods is competitive, the representative firm's profit must be zero, i.e. the object function (17) is equal to 0. Combine with the result of (18) and express $Y_t(i)$ in terms of the reference good z

$$
\begin{aligned}
P_t Y_t &= \int_0^1 P_t(i) Y_t(i) di, \\
P_t P_t(z)^{\epsilon} Y_t(z) \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} &= \int_0^1 P_t(i) \left[\frac{P_t(z)}{P_t(i)}\right]^{\epsilon} Y_t(z) di, \\
P_t P_t(z)^{\epsilon} Y_t(z) \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{\epsilon}{\epsilon-1}} &= P_t(z)^{\epsilon} Y_t(z) \int_0^1 P_t(i)^{1-\epsilon} di, \\
P_t &= \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.
\end{aligned}
$$

Therefore the price for the final goods P_t can be expressed as a weighted average of prices of the intermediate goods, which is sometimes called the *price index* for the intermediate goods.

Then by rearranging the equation (18) and applying the price index, the demand for the intermediate good z is

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t. \tag{19}$$

Since P_t and Y_t are both exogenous for an individual final goods producer, the demand for an intermediate good z is solely determined by its price asked by the upstream (intermediate goods) producers. Remeber that ϵ defines the elasticity of substitution in the production function, therefore the higher ϵ is, the easier it is to substitute the input z with the others, hence the more sensitive the demand $Y_t(z)$ is in responding its price $P_t(z)$.

2.3.2 The Wholesale Firm

The intermediate products, used as input for the final goods production, are manufactured by the wholesale firms. In the wholesale sector there is a continuum of *monopolistically competitive* firms owned by the consumers (There is no loss of generality to assume that merely the wholesale firms, instead of all three types of the firms, are owned by the consumers. The reason is that the wholesale sector is the only part yielding the strictly positive profit, which enters the representative agent's resource constraint). These firms are indexed by z and the measure of them is normalized to 1. The firms are monopolistically competitive in the sense that each one of them faces the downward sloping demand curve (19) for its product that is specific and an imperfect substitute for the other goods. Therefore these firms have some monopolistic power in their price decisions.

I. Factor Demand and Marginal Cost

A representative wholesale firm z follows the neoclassical production function

$$Y_t(z) = A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha}$$

in which A_t is the exogenous parameter for technological progress, and the capital stock $K_t(z)$ and employed labor $N_t(z)$ are used as inputs.

Similar as before, the representative wholesale firm's problem is to maximize its profit (PMP), which is equivalent to minimize its production cost with some threshold level of output $Y_t(z)$ as given (EMP). The cost can be decomposed into two parts: One is the wage paid for the labor, the other is the rent paid for the capital at the rate Z_t. In summary,

$$\begin{aligned} \min_{N_t(z),K_t(z)} \; & \frac{W_t N_t(z)}{P_t} + Z_t K_t(z), \\ s.t.\; & A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \geq Y_t(z). \end{aligned}$$

Set up the Lagrangian

$$\mathscr{L}_t = \frac{W_t N_t(z)}{P_t} + Z_t K_t(z) + \lambda_t \left[Y_t(z) - A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \right],$$

and the first order conditions are

$$\frac{\partial \mathscr{L}_t}{\partial N_t(z)} = \frac{W_t}{P_t} - \lambda_t A_t \alpha N_t(z)^{\alpha-1} K_t(z)^{1-\alpha} = 0,$$
$$\frac{\partial \mathscr{L}_t}{\partial K_t(z)} = Z_t - \lambda_t A_t (1-\alpha) N_t(z)^{\alpha} K_t(z)^{-\alpha} = 0.$$

Since the inequality constraint is binding at the optimum, i.e.

$$A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} = Y_t(z), \tag{20}$$

therefore the first order conditions can be rewritten as

$$\lambda_t = \frac{W_t}{P_t A_t \alpha N_t(z)^{\alpha-1} K_t(z)^{1-\alpha}} = \frac{W_t}{P_t} \frac{N_t(z)}{\alpha Y_t(z)} = MC_t, \tag{21}$$
$$\lambda_t = \frac{Z_t}{A_t (1-\alpha) N_t(z)^{\alpha} K_t(z)^{-\alpha}} = \frac{Z_t K_t(z)}{(1-\alpha) Y_t(z)} = MC_t, \tag{22}$$

note that the Lagrange multiplier, or the shadow price, λ_t reflects the impact on total cost if we relax the inequality constraint by producing one unit more good z. Therefore λ_t is just the real marginal cost of the representative firm, denoted by MC_t.

Solve for $N_t(z)$ from (21), $K_t(z)$ from (22) and insert these two results into (20), one can get

$$MC_t = \frac{1}{A_t} \left(\frac{W_t}{P_t \alpha} \right)^{\alpha} \left(\frac{Z_t}{1-\alpha} \right)^{1-\alpha}. \tag{23}$$

Since the market wage rate W_t, market rental rate Z_t and technological A_t are exogenous and constant across the firms, by (23) the marginal cost should be the same for all the firms (which is the natural result of our assumption on constant return to scale technology and perfect factor mobility, i.e. free market for capital and labor). That's why we neglect the index z for MC_t.

From (21) rearrange to get

$$P_t(z) \alpha Y_t(z) = \frac{P_t(z)}{MC_t} \frac{W_t}{P_t} N_t(z) = (1+\mu_t) W_t N_t(z) \tag{24}$$

in which μ_t is the markup featuring the gap between the firm's marginal revenue (the price $P_t(z)$ which the wholesale firm z asks) and the marginal cost (which is the real marginal cost priced by the price level in the economy), such that

$$1 + \mu_t = \frac{P_t(z)}{P_t MC_t}. \tag{25}$$

From (22) rearrange to get

$$P_t(z)(1 - \alpha)Y_t(z) = \frac{P_t(z)}{MC_t P_t} P_t Z_t K_t(z) = (1 + \mu_t) P_t Z_t K_t(z). \tag{26}$$

Equations (24) and (26) are pretty similar to what we got in growth models. The left-hand sides, $P_t(z)\alpha Y_t(z)$ and $P_t(z)(1-\alpha)Y_t(z)$ are the market values of the shares of output associated with labor and capital respectively; and the right-hand sides, $W_t N_t(z)$ and $P_t Z_t K_t(z)$ are the nominal costs paid to the factors. The only difference is that these two equations are adjusted by the markup, $1+\mu_t$, implying that the firms adjust each input to the point where the marginal product is still higher than the factor price in order to secure the markup μ_t.

Note that by symmetry in equilibrium all the wholesale firms charge the same price, then $P_t(z) = P_t$, $\forall z \in [0, 1]$ by the definition of P_t. In this case the gross markup μ_t becomes

$$1 + \mu_t = \frac{1}{MC_t} \tag{27}$$

which is a constant for all the firms.

To see exactly how large μ_t is, start from the representative firm's profit maximization problem. In our model the firm's real marginal cost is defined by equation (23), which is a complicated combination of the real labor and capital costs which are exogenous to individual firms. To make it simpler, we define the firm's nominal marginal cost as $MC_t^n = P_t MC_t$. Then the firm's problem is to maximize its profit by asking the price level $P_t(z)$ for its output $Y_t(z)$, which has to match the downstream firms' demand (19)

$$\max_{P_t(z)} \left[P_t(z) - MC_t^n\right] Y_t(z), \tag{28}$$

$$s.t.\ Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t. \tag{29}$$

The first order condition gives

$$(1 - \epsilon)\left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t + \epsilon MC_t^n \frac{1}{P_t(z)} \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t = 0,$$

and the optimal price $P_t(z)$ can be solved

$$P_t(z) = \frac{\epsilon}{\epsilon - 1} MC_t^n = (1 + \mu) MC_t^n. \tag{30}$$

This equation explains where the price markup comes from. μ can be expressed as

$$\mu = \frac{1}{\epsilon - 1},$$

which relates to the inverse of ϵ, the elasticity of substitution of the final goods producers' production function: The lower ϵ is, the harder it is for the downstream firms, i.e. the final goods producers, to substitute one input with the others, therefore the upstream firms, i.e. the intermediate goods producers, have more monopolistic power on pricing, implying a higher markup.

II. Staggered Price Adjustment and Optimal Price Setting

Instead of assuming that the wholesale firms set their prices by immediately responding the shocks in the marginal cost, we introduce the nominal rigidity here by assuming that the firms adjust their prices in a staggered manner. Prices are adjusted à la Calvo, which assumes that the wholesale firms adjust their prices infrequently and that opportunities to adjust arrive as an exogenous Poisson process. In each period a firm adjusts its price with a constant probability $1-\theta$ and keeps it price fixed with probability θ. Therefore by the law of large number, in each period there is a share $1-\theta$ of the firms adjusting their prices and the expected time between a firm's two successive adjustments is $\frac{1}{1-\theta}$ periods — Because these adjustment opportunities occur randomly, the interval between price changes for an individual firm is a random number.

For a firm setting its price $P_t(z)$ at period t, the optimal level of $P_t(z)$, denoted by $P_t^*(z)$, maximizes the firm's expected profit. Note that the firm's profit under $P_t^*(z)$, denoted by $\Pi_z(P_t^*(z))$, is only achieved for the future periods in which $P_t^*(z)$ is maintained: For period $t+1$ the probability that $P_t^*(z)$ is maintained is θ, therefore the firm's expected profit under $P_t^*(z)$ in period $t+1$ is $\theta\Pi_{z,t+1}(P_t^*(z))$. For the same reason the firm's expected profit under $P_t^*(z)$ in period $t+i$ is $\theta^i\Pi_{z,t+i}(P_t^*(z))$, therefore in present value the firm's expected profit under $P_t^*(z)$ after the price adjustment is

$$\Pi_z(P_t^*(z)) = \sum_{i=0}^{+\infty}\theta^i E_t\left[\frac{1}{R_{t,t+i}}\Pi_{z,t+i}(P_t^*(z))\right]$$

in which $\Pi_{z,t+i}(P_t^*(z))$ can be expressed as the product of the real marginal profit and the the firm's output

$$\Pi_{z,t+i}(P_t^*(z)) = \frac{P_t^*(z) - P_{t+i}MC_{t+i}}{P_{t+i}}Y_{t+i}(z)$$

with P_{t+i} and MC_{t+i} denoting the levels of the price index and the marginal cost in period $t+i$ (remember that P_{t+i} and MC_{t+i} are exogenous for an individual firm, as we have shown before).

Furthermore, the output level of the firm, $Y_{t+i}(z)$, is simply determined by the demand curve of the downstream, i.e. the final goods, producers, as written in equation (19). Then we can finalize the representative firm's problem on optimal price setting as following

$$\begin{aligned}
&\max_{P_t(z)} \Pi_z(P_t(z)) = \sum_{i=0}^{+\infty} \theta^i E_t \left[\frac{1}{R_{t,t+i}} \frac{P_t(z) - P_{t+i} MC_{t+i}}{P_{t+i}} Y_{t+i}(z) \right], \\
&s.t.\ Y_{t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i}.
\end{aligned}$$

To solve it, simply insert the equality constraint into the object function

$$\max_{P_t(z)} \Pi_z(P_t(z)) = \sum_{i=0}^{+\infty} \theta^i E_t \left\{ \frac{1}{R_{t,t+i}} \frac{P_t(z) - P_{t+i} MC_{t+i}}{P_{t+i}} \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} \right\},$$

and obtain its first condition with respect to $P_t(z)$

$$\begin{aligned}
\frac{\partial \Pi_z(P_t(z))}{\partial P_t(z)} &= E_t \left(\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} \left\{ \frac{1}{P_{t+i}} \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} + \frac{P_t(z) - P_{t+i} MC_{t+i}}{P_{t+i}} (-\epsilon) \left[\frac{P_t(z)}{P_{t+i}} \right]^{-\epsilon - 1} \frac{1}{P_{t+i}} Y_{t+i} \right\} \right) \\
&= 0.
\end{aligned}$$

Note that we have already shown

$$\left[\frac{P_t(z)}{P_{t+i}} \right]^{-\epsilon} Y_{t+i} = Y_{t+i}(z)$$

in equation (19), the first order condition can be rewritten as

$$\begin{aligned}
E_t \left(\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} \left\{ \frac{1}{P_{t+i}} Y_{t+i}(z) + \frac{P_t(z) - P_{t+i} MC_{t+i}}{P_{t+i}} (-\epsilon) \left[\frac{P_t(z)}{P_{t+i}} \right]^{-1} \frac{1}{P_{t+i}} Y_{t+i}(z) \right\} \right) &= 0, \\
E_t \left(\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} \left\{ \frac{1}{P_{t+i}} Y_{t+i}(z) - \epsilon \frac{1}{P_{t+i}} Y_{t+i}(z) + \epsilon MC_{t+i} \frac{Y_{t+i}(z)}{P_t(z)} \right\} \right) &= 0, \\
-(\epsilon - 1) E_t \left[\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} \frac{1}{P_{t+i}} Y_{t+i}(z) \right] + \frac{\epsilon}{P_t(z)} E_t \left[\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} MC_{t+i} Y_{t+i}(z) \right] &= 0,
\end{aligned}$$

then $P_t(z)$ can be solved

$$P_t(z) = \frac{\epsilon}{\epsilon - 1} \frac{E_t \left[\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} MC_{t+i} Y_{t+i}(z) \right]}{E_t \left[\sum_{i=0}^{+\infty} \frac{\theta^i}{R_{t,t+i}} \frac{1}{P_{t+i}} Y_{t+i}(z) \right]}$$

$$= (1+\mu)\frac{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}MC_{t+i}Y_{t+i}(z)\right]}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\frac{1}{P_{t+i}}Y_{t+i}(z)\right]},$$

which says that the optimal price depends on the weighted average of the marginal cost adjusted by the term $\frac{\epsilon}{\epsilon-1}$. Using a similar definition as before, we define μ as the firm's markup such that (note that $\epsilon > 1$)

$$1+\mu = \frac{\epsilon}{\epsilon-1} = 1+\frac{1}{\epsilon-1}.$$

To see it cleary what the optimal price depends on, again using the fact that

$$Y_{t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon} Y_{t+i}$$

to rewrite the expression for $P_t(z)$

$$\begin{aligned}
P_t(z) &= (1+\mu)\frac{E_t\left\{\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}MC_{t+i}\left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon}Y_{t+i}\right\}}{E_t\left\{\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\frac{1}{P_{t+i}}\left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon}Y_{t+i}\right\}}\\
&= (1+\mu)\frac{E_t\left\{\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}MC^n_{t+i}\frac{1}{P_{t+i}}\left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon}Y_{t+i}\right\}}{E_t\left\{\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\frac{1}{P_{t+i}}\left[\frac{P_t(z)}{P_{t+i}}\right]^{-\epsilon}Y_{t+i}\right\}}\\
&= (1+\mu)\frac{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}MC^n_{t+i}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}\\
&= (1+\mu)\frac{\sum_{i=0}^{+\infty}\left\{E_t\left[\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]MC^n_{t+i}\right\}}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}\\
&= (1+\mu)\sum_{i=0}^{+\infty}\psi_{t+i}MC^n_{t+i}
\end{aligned}$$

in which for simplicity we define ψ_{t+i} as

$$\psi_{t+i} = \frac{E_t\left[\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}$$

and again the *nominal* marginal cost MC^n_t is defined as

$$MC_t^n = P_t MC_t.$$

Notice that all the components in the expression for ψ_{t+i} are exogenous for an individual firm, therefore essentially the optimal price equals the markup times a weighted average of expected future nominal marginal cost. The weights depends on how the firm discounts future cash flows in each period $t+i$ (taking into account that the price remains fixed in $t+i$), and also the relative proportion of revenues expected in each period. The latter hinges on the future expected values of *aggregate* variables Y_{t+i} and P_{t+i}.

To see it explicitly how the price decision depends on θ, the measure determining the price stickiness, pick up an intermediate step as following

$$P_t(z) = (1+\mu)\frac{\sum_{i=0}^{+\infty}\left\{E_t\left[\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]MC_{t+i}^n\right\}}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}. \tag{31}$$

Obviously when $\theta = 0$ the problem is degenerated to the original flexible price decision problem without any stickiness in pricing behavior, and the equation (31) turns out to be

$$P_t(z)|_{\theta=0} = (1+\mu)MC_t^n \tag{32}$$

which is exactly what we got in equation (30).

In equilibrium, in any period t by the law of large number there are a share $1-\theta$ of the firms adjusting their prices following the optimal strategy

$$P_t^*(z) = (1+\mu)\sum_{i=0}^{+\infty}\psi_{t+i}MC_{t+i}^n,$$

and a share θ of the firms which do not adjust their prices, hence simply continue with the price level in the past period P_{t-1}. Then in this scenario the price index for period t is computed by integrating these two types of firms

$$\begin{aligned}
P_t &= \left[\int_0^1 P_t(z)^{1-\epsilon}dz\right]^{\frac{1}{1-\epsilon}}\\
&= \left[\int_0^\theta P_{t-1}^{1-\epsilon}dz + \int_\theta^1 P_t^{*1-\epsilon}dz\right]^{\frac{1}{1-\epsilon}}\\
&= \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)P_t^{*1-\epsilon}\right]^{\frac{1}{1-\epsilon}}
\end{aligned}$$

— Note that the measures for these two types of firms are θ and $1-\theta$, respectively. And sometimes it's useful to use the equation in terms of inflation:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon},$$
$$1 = \theta \pi_t^{\epsilon-1} + (1-\theta) \tilde{P}_t^{*1-\epsilon}$$

in which $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate, and $\tilde{P}_t^* = \frac{P_t^*}{P_t}$ is the relative price between the optimally adjusted price and the price level of period t.

2.3.3 The Capital Producers

After production of final goods each period, *competitive* capital producers make new capital goods. Capital producer j purchases a certain amount of the final goods to use as materials input $I_t(j)$ and the capital $K_t(j)$ as factor of production. They rent the capital at the real rate of Z_t^k after it has been used by the final goods producers to produce the final output within the period. They sell new capital produced at the real market price Q_t.

The production function for new capital $Y_t^k(j)$ is given by

$$Y_t^k(j) = \phi\left(\frac{I_t(j)}{K_t(j)}\right) K_t(j) \tag{33}$$

in which $\phi\left(\frac{I_t(j)}{K_t(j)}\right)$ is the function of the ratio of input and capital such that $\phi : \mathbb{R} \to \mathbb{R}_+$ with $\phi'(\cdot) > 0$, $\phi''(\cdot) < 0$ and $\phi\left(\frac{I^*}{K^*}\right) = \frac{I^*}{K^*}$, I^* and K^* being the steady state input and capital (the reason behind such assumptions will be made clear in the end of this section). Note that the production function exhibits the properties of constant return to scale as well as diminishing marginal product to the inputs.

A representative capital producer's problem is to maximize its real profit

$$\max_{I_t(j), K_t(j)} \Pi_j = Q_t \phi\left(\frac{I_t(j)}{K_t(j)}\right) K_t(j) - I_t(j) - Z_t^k K_t(j). \tag{34}$$

The first order conditions are

$$\frac{\partial \Pi_j}{\partial I_t(j)} = Q_t \phi'\left(\frac{I_t(j)}{K_t(j)}\right) - 1 = 0, \tag{35}$$
$$\frac{\partial \Pi_j}{\partial K_t(j)} = -Q_t \phi'\left(\frac{I_t(j)}{K_t(j)}\right) \frac{I_t(j)}{K_t(j)} + Q_t \phi\left(\frac{I_t(j)}{K_t(j)}\right) - Z_t^k = 0. \tag{36}$$

Equation (35) shows that

$$
\begin{aligned}
Q_t &= \frac{1}{\phi'\left(\frac{I_t(j)}{K_t(j)}\right)}, \\
\frac{dQ_t}{d\left(\frac{I_t(j)}{K_t(j)}\right)} &= -\left[\phi'\left(\frac{I_t(j)}{K_t(j)}\right)\right]^{-2} \phi''\left(\frac{I_t(j)}{K_t(j)}\right) \\
&> 0,
\end{aligned}
$$

meaning that Q_t increases with $\frac{I_t(j)}{K_t(j)}$, which is the same as in Tobin's q theory.

Equation (36) shows that

$$
Q_t \left[\phi\left(\frac{I_t(j)}{K_t(j)}\right) - \phi'\left(\frac{I_t(j)}{K_t(j)}\right)\frac{I_t(j)}{K_t(j)}\right] = Z_t^k. \tag{37}
$$

If equation (37) is valued in the steady state, then it becomes

$$
\begin{aligned}
& Q^* \left[\phi\left(\frac{I^*}{K^*}\right) - \phi'\left(\frac{I^*}{K^*}\right)\frac{I^*}{K^*}\right] \\
&= Q^* \left[\frac{I^*}{K^*} - \frac{I^*}{K^*}\right] \\
&= 0 \\
&= Z^{k*}.
\end{aligned}
$$

Therefore as a reasonable approximation we simply set $Z_t^k = 0$, as long as we are only interested in the local behavior of the economy around the steady state.

It follows that the steady state value of Q_t is 1 from equation (35). Since the capital producers are competitive, the representative firm's profit must be zero. Then the firm's profit function (34) shows that in the steady state the input I^* leads to an output I^*, which is then sold at the real price $Q^* = 1$ and becomes the increment in the representative consumer's capital stock, i.e. the investment in the economy is tranformed into capital in a manner of one for one. However, when the economy is off equilibrium, the concavity of the production function (33) adds a convex cost of adjusting capital stock, introducing the real rigidity in the model (to be explained in Section 3.2).

As is shown in Figure 2, in the steady state the investment I^* is transformed into $\phi\left(\frac{I^*}{K^*}\right)K^* = \frac{I^*}{K^*}K^* = I^*$ units of new capital, making the level of total capital stock K^* unchanged. Now suppose that the investment increases to $\tilde{I} > I^*$. Since $\phi'(\cdot) > 0$,

$$
\phi\left(\frac{\tilde{I}}{K^*}\right)K^* > \phi\left(\frac{I^*}{K^*}\right)K^* = I^*,
$$

meaning that the capital stock will increase. However, since $\phi''(\cdot) < 0$,

$$\phi\left(\frac{\tilde{I}}{K^*}\right)K^* < \frac{\tilde{I}}{K^*}K^* = \tilde{I},$$

meaning that the investment input $\tilde{I}$ is transformed into less than $\tilde{I}$ units of new capital, generating a cost of

$$\tilde{I} - \phi\left(\frac{\tilde{I}}{K^*}\right)K^*,$$

which is convex in $\frac{I}{K}$, in adjusting the level of capital stock from its equilibrium value. Therefore establishing the sector of capital producers in our model achieves the same goal of introducing capital adjustment cost as that of the standard Tobin's q model.

2.4 The Government

The government contributes some expenditure G_t in each period to this economy, and G_t is financed by money printing and lump-sum tax TR_t (If $TR_t > 0$ then it's a lump-sum transfer). The government budget constraint is

$$\frac{M_t - M_{t-1}}{P_t} = G_t + TR_t. \tag{38}$$

In addition, the government implements its monetary policy through some *interest rule* by controlling the nominal interest rate R_t^n in each period

$$R_t^n = R^{n*}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y^*}\right)^{\gamma_y} e^{\epsilon_t^r},$$

in which R^{n*} is some anchor value for the nominal interest rate, ϵ_t^r is a ramdom variable to capture the uncertaity associated with the monetary rule, and the government chooses the parameters $\gamma_\pi > 0$ and $\gamma_y > 0$ in response to the inflation and the output gap respectively. The government is *aggressive* about the inflation rate if $\gamma_\pi > 1$, which is termed as the Taylor's Principle (Taylor, 1993). In the next chapter, we'll explain this principle in detail.

3 Monopolistic Competition and Rigidities Revisited

Now we make some addendums for several issues that have been discussed so far. These are not directly associated the model we are going on with, but they explain why we adopt some exotic assumptions and how they relate with the results we desire.

The assumption of monopolistic competition introduces the first distortion into the frictionless models we have studies in the past two months. Overall the notion of monopolistic competition works as a bridge in the entire model: On one hand it leads to interesting macroeconomic consequences, paving the path to the prospective intervention; on the other hand, before adding the rigidities in the price adjustment we need a model that explains how the firms choose their optimal prices and what takes place when one deviates from such optimum.

3.1.1 Inefficiencies from Monopolistic Competition

In order to see the direct effects of monopolistic competition, from now on we concentrate on the static problem in which the monopolistically competitive wholesale firms set their one-shot optimal prices, in the absence of nominal rigidities. Therefore in equilibrium the optimality conditions are just the static version of equations (21) and (22)

$$\frac{W}{PA\alpha N(z)^{\alpha-1}K(z)^{1-\alpha}} = MC, \tag{39}$$

$$\frac{Z}{A(1-\alpha)N(z)^{\alpha}K(z)^{-\alpha}} = MC. \tag{40}$$

Apply the definition of μ, equation (27) as well as the production function, one can see that

$$\frac{W}{P} = \frac{1}{1+\mu}\frac{\partial Y(z)}{\partial N(z)}, \tag{41}$$

$$Z = \frac{1}{1+\mu}\frac{\partial Y(z)}{\partial K(z)}. \tag{42}$$

Equation (41) means that under monopolistic competition the real wage is lower than the marginal product of labor, and equation (42) means that the real rental rate of the capital is lower than the capital's marginal product. These facts imply that the equilibrium outcome is inferior to the social optimal allocation, and Problem 4 from Problem Set 4 asks the readers to solve for these levels.

Further, combine the intratemporal optimality conditions (1) and (2) one can get

$$-\frac{\frac{\partial u_t}{\partial N_t}}{\frac{\partial u_t}{\partial C_t}} = \frac{W_t}{P_t}, \tag{43}$$

meaning that the marginal rate of substitution between labor and consumption equals the real wage. However equation (41) already shows that under monopolistic competition the real wage is below the social optimal level, implying that the representative agent's intratemporal decisions are also distorted.

3.1.2 Menu Cost

The assumption of monopolistic competition may itself lead to a theory of nominal rigidity, which was popular before Calvo's staggering pricing was widely applied. Consider an economy of monopolistically competitive firms (the wholesale firms in our model) facing downward sloping demand curves and being initially at the equilibrium level such that the marginal revenue equals the marginal cost for each firm. Figure 3 shows the optimal price level of a representative firm: Given the demand curve D and the corresponding marginal revenue curve MR the optimal output q is achieved where MR and the marginal cost curve, MC, cross each other, and the optimal price is set by $p = D^{-1}(q)$ as point A shows.

Then suppose now there is an unexpected fall in aggregate demand Y_t, by equation (19) this implies a proportional drop in the representative firm's demand which shifts D curve inward to D' and MR curve to MR', therefore the new optimal strategy for the firm becomes (p', q') as point C in the figure.

Now the question is, how high the incentive it is for the firm to adjust its price level from p to p'? The motivation behind this question is that if the incentive is small enough, even a minor exogenous cost associated with such price adjustment may deter the setting of the new price!

Suppose that firm simply keeps the old price p, then the new output is determined by the new demand curve, as point B shows. Notice that the profit for the firm is just the area between MR and MC curves, the loss from keeping the old price, or the incentive to adjust the price, is the red triangle area (denoted by $\Delta\Pi(z)_1$ for simplicity) — It is indeed small! And the area would be even smaller, if the demand elasticity becomes higher, i.e. when the firm faces a flatter D curve.

Given that the firm's incentive for price adjustment in response to an aggregate demand shock is small, and smaller when the consumers are more sensitive to the price change (under a flatter D curve), now we assume that there is a *menu cost* c^{MENU} associated with the price change (such cost may be as small as printing a new version of your menu for the new prices). Then if the gain from any price change is no higher than the menu cost, $\Delta\Pi(z)_1 \leq c^{MENU}$, such price adjustment would be deterred.

Essentially the rigidity in the price setting in the presence of menu cost reflects the firms' coordination failure. Note that in a wholesale firm's profit maximization problem, (28) with (29), what the firm takes into account are his *individual* demand curve $Y_t(z)$ and the nominal marginal cost, but these two factors are governed by the economy-wide parameters, Y_t and P_t, which are only influenced by the aggregate outcome of all the firms' behavior that is exogenous to an individual firm. This implies that under monopolistic competition the firms' pricing decisions have externalities, and those externalities work through the aggregate demand. Therefore such externality is named as *aggregate demand externality* by Blanchard and Kiyotaki (1987).

To make it clear, we start from FIGURE 3. The individual firm's incentive to reduce its price to p' is small because its demand curve D is shifted inwards to D', making its profit gain from reducing price (the area of the red triangle, $\Delta\Pi(z)_1$) too little. Surely the firm would prefer to come back to the original D curve, but this can only be achieved via the coordination of all the firms. A further investigation shows that the firms can almost restore the original aggregate demand, hence each individual demand curve, by cutting their prices a little bit from p to p'' as FIGURE 4 shows [1]. A firm's gain from this price cut is the area shaded by the horizontal green lines (denoted by $\Delta\Pi(z)_2$), which is much larger than $\Delta\Pi(z)_1$ and quite likely that $\Delta\Pi(z)_1 \leq c^{MENU} < \Delta\Pi(z)_2$ — All the firms are better off by such coordinated price cut.

But is it possible that the price cut is achieved by each firm? Note that the demand curve faced by an individual firm is D' instead of D'' which is realized only *after* the *coordinated* price cut. Therefore if one firm unilaterally adjusts its price from p to p'' the expected profit gain is merely the area shaded by the vertical blue lines, denoted by $\Delta\Pi(z)_3$. Obviously $\Delta\Pi(z)_3 < \Delta\Pi(z)_1 \leq c^{MENU}$, and nobody would initiate the price cut! The coordination would never work, although it makes every firm better off.

3.2 Incomplete Nominal Adjustment and Replacement Cost: The Needs for Rigidities

Now we have introduced distortions to our model economy via monopolistic competition, however, this doesn't mean that money is no longer neutral. If firms are flexible and complete in changing their prices (in the absence of any resources of price stickiness, e.g. menu cost), then the monetary shock would simply lead to proportioanal changes in the nominal wage and price levels; and the real variables, such as output and the real wage, are unaffected — Monetary policy still has no real effect.

[1] However, this is not easily seen in current stage because we haven't yet introduced general equilibrium. The reasoning is sketched as following: In equilibrium the government's profit from printing money is balanced by its spending on transfer and other expenditure, as equation (38) shows. And also in equilibrium the borrowing and lending should offset each other, there would be no net debt. Therefore the representative agent's budget constraint becomes

$$\frac{WN}{P} + ZK + \Pi - G + Q(1-\delta)K = C + QK,$$

meaning that an unexpected drop in the price level P increases consumption level C. The economy's resource constraint (which will be clear in the next chapter) requires that the aggregate output be identical to the aggregate expenditure, such that

$$Y = C + I + G.$$

This implies that an unexpected increase in aggregate consumption increases the level of Y, i.e. the aggregate demand.

To see this, suppose that there is an expected increase in money supply. If the price adjustment is complete, the firms will adjust the nominal wage and nominal prices to maintain the equilibrium — Note the optimality conditions (1) – (5), (19), (21) and (22) only depend on the real variables. And the firms' real marginal cost, defined in equation (23)

$$MC = \frac{1}{A}\left(\frac{W}{P\alpha}\right)^{\alpha}\left(\frac{Z}{1-\alpha}\right)^{1-\alpha}. \tag{44}$$

will continue to stay at a constant; therefore the firms' markup doesn't change, and no real effect will take place.

Therefore if money matters, or monetary shock have real effects, the price adjustment cannot be complete. In order to obtain the results we desire, there might be two kinds of approaches:

- Introducing *real rigidities*, i.e. the firms have the flexibility in changing nominal wages and prices, but there are some real costs associated with price settings or rigidities in the adjustments of the real variables, making the equilibrium outcome deviated from the optimal allocation. For example,
  - Menu cost as in Section 3.1.2, deterring the price adjustments, e.g. Akerlof and Yellen (1985), Mankiw (1985);
  - Sticky information, the cost in acquiring information deters the price adjustments, e.g. Mankiw and Reis (2002);
  - Replacement cost, as in Section 2.3.3, the cost associated with investment introduces the persistence in capital adjustments, e.g. Woodford (2003);
  - Habit, such that the consumers have some persistence to change their consumption over time, e.g. Ravn, Schmitt-Grohé and Uribe (2006);
  - Imperfections in labor market;
  - ……
- Introducing *nominal rigidities*, i.e. the firms fail to adjust nominal wages and prices immediately and completely. For example,
  - Wage rigidity, i.e. wages are set at some early period and are unresponsive to shocks in the near future, e.g. Taylor (1979, 1980). The effect can be immediately seen from equation (44): If one firm's nominal wage W doesn't comove with the price level P, its real marginal cost changes, leading to the changes in its profit and output levels;
  - Monopolistic competition and price stickiness à la Calvo (1983), as in Section 2.3.2. The effect can be immediately seen from equation (25): If one firm's nominal price $P(z)$ doesn't comove with the price level P, its markup changes, leading to the changes in its profit.

4 Readings

Blanchard and Fischer (1989), Chapter 8.1; Blanchard and Kiyotaki (1987); Galí (2008), Chapter 3.

5 Bibliographic Notes

The first dynamic stochastic general equilibrium based new keynesian monetary model with nominal rigidities is built up in Yun (1996), and Schmitt-Grohé and Uribe (2005) integrates all notable nominal as well as real rigidities in a single framework. Gertler (2003) is a widely adopted textbook approach to Yun (1996), and Galí (2008) is one of the latest textbooks on this issue.

The sector structure of the firms largely follows Chari, Kehoe and McGrattan (2000). The capital accumulation process is a simplified and augmented version of Schmitt-Grohé and Uribe (2005) as well as Woodford (2003), Chapter 4. In these two works the investment cost is explicitly written in the household's budget constraint, while in our model this part of cost is separated by adding the capital producers into the firms, hoping to make the model cleaner and more elegant.

The discussion on menu cost is based on Romer (1993), and the idea was proposed in a number of works, such as Akerlof and Yellen (1985), Mankiw (1985). The discussion on aggregate demand externalities is based on Blanchard and Kiyotaki (1987), Blanchard and Fischer (1989), Ball and Romer (1991) and Romer (2006). Walsh (2010) Chapter 5 provides an excellent review on different approaches to adding nominal as well as real rigidities in monetary models.

Some other interesting works are already cited in the progress of each section.

6 Exercises

6.1 Dixit-Stiglitz Indices for Continuous Commodity Space

(Dixit and Stiglitz, 1977) Consider a one-person economy. Mr. Rubinson Crusoe is the only agent in this economy, consuming a continuum of commodities $i \in [0, 1]$. Suppose that the consumption index C of him is defined as

$$C = \left[\int_0^1 Z_i^{\frac{1}{\eta}} C_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}$$

in which C_i is the consumption of good i and Z_i is the taste shock for good i. Suppose that Crusoe has an amount of endowment Y to spend on goods with exogenously given price tags. Therefore the budget constraint is

$$\int_0^1 P_i C_i di = Y.$$

a)[A] Find the first-order condition for the problem of maximizing C subject to the budget constrain. Solve for C_i in terms of Z_i, P_i and the Lagrange multiplier on the budget constraint.

b)[B] Use the budget constraint to find C_i in terms of Z_i, P_i and Y.

c)[B] Insert the result of **b)** into the expression for C and show that $C = \frac{Y}{P}$, in which

$$P = \left(\int_0^1 Z_i P_i^{1-\eta} di \right)^{\frac{1}{1-\eta}}.$$

d)[B] Use the results in **b)** and **c)** to show that

$$C_i = Z_i \left(\frac{P_i}{P}\right)^{-\eta} \left(\frac{Y}{P}\right).$$

Interpret this result.

6.2 Price Setting with Differentiated Goods

Consider a representative agent with utility function

$$U = \left(\sum_{i=1}^{m} C_i^{\gamma} \right)^{\frac{\alpha}{\gamma}} \left(\frac{M}{P}\right)^{1-\alpha} - N^{\beta}, \text{ with } 0 < \gamma < 1, 0 < \alpha < 1, \beta > 1.$$

Assume that firm's profits are distributed to consumers, but a single consumer's decision has no impact on these profits. Thus, profit income is taken as exogenous by consumers.

a)[B] Derive the demand functions for commodities C_i and for money M and the supply for labor N. To ease your calculations, use aggregate indices for consumption and prices:

$$C = \left(\sum_{i=1}^{m} C_i^{\gamma} \right)^{\frac{1}{\gamma}}, P = \left(\sum_{i=1}^{m} P_i^{-\frac{\gamma}{1-\gamma}} \right)^{-\frac{1-\gamma}{\gamma}}.$$

b)[C] Assume that firms produce goods with production function $C_i = \theta N_i$, where N_i is the labor input of firm i. Labor is homogeneous and the labor market is competitive. Firms are setting prices P_i in order to maximize profits. Show that equilibrium prices are above marginal costs (Assume that firms are small to the extend that a single firm's decisions has no impact on average income of households).

c)[B] Show that equilibrium levels of production and employment are below the efficient levels.

d)[C] Following Blanchard & Kiyotaki (1987), explain how menu costs can prevent price adjustments to monetary expansion and how this influences overall efficiency.

6.3 Menu Cost and Nominal Price Rigidity

A representative monopolistically competitive firm sells its output for a nominal price P_i. It faces the demand function

$$Y_i = \left(\frac{P_i}{P}\right)^{-\epsilon} D \text{ with } \epsilon > 1$$

in which P is the general price level of the economy and D is an aggregate demand parameter (both of them are exogenous to the firm). There is only one productive input, labor L_i , which is used according to the production function

$$Y_i = L_i^{\frac{1}{\beta}} \text{ with } \beta > 1.$$

The firm pays workers an exogenous nominal wage w.

a)[A] Explain the parameter β. What is the economic interpretation of the condition $\beta > 1$?

b)[B] Draw a diagram with the demand function, the marginal revenue function and the marginal cost function of the firm. Determine the firms optimal price and quantity.

c)[B] Use your diagram to demonstrate the response of the optimal relative price to a fall in D.

d)[B] Use this example to explain the menu cost theory of nominal price rigidity. Which factors determine the degree of rigidity?

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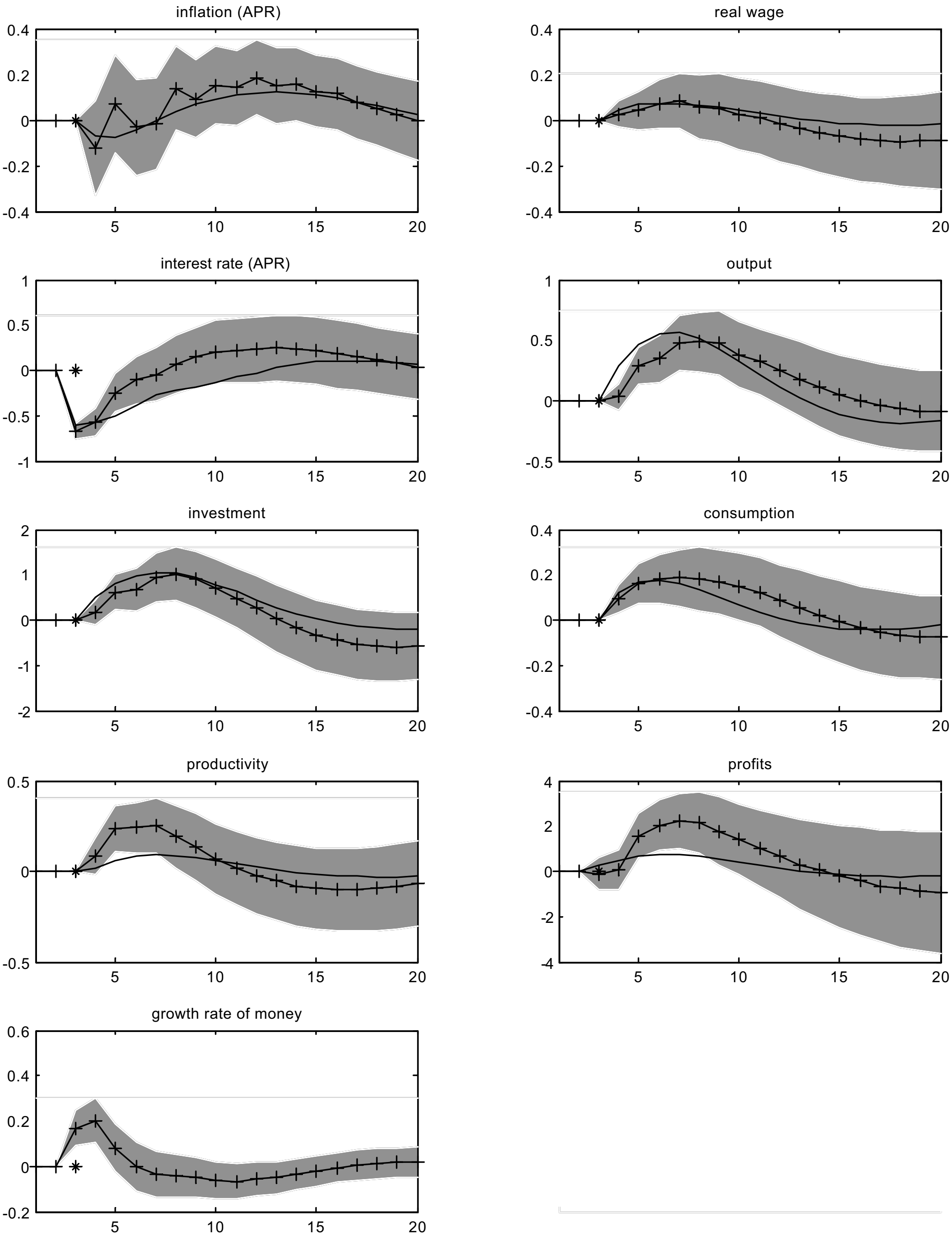

Fig. 1. VAR-Based Impulse Responses. Solid lines — CEE's benchmark model (which is similar as ours, with an extra real ridigity in labor supply) impulse responses; Solid lines with + — VAR-based impulse responses; Grey area — 95% confidence intervals about VARbased estimates. Units on horizontal axis — quarters. * — indicates the period of policy shock. Vertical axis indicate deviations from unshocked path. Inflation, money growth, interest rate — annualized percentage points (APR).

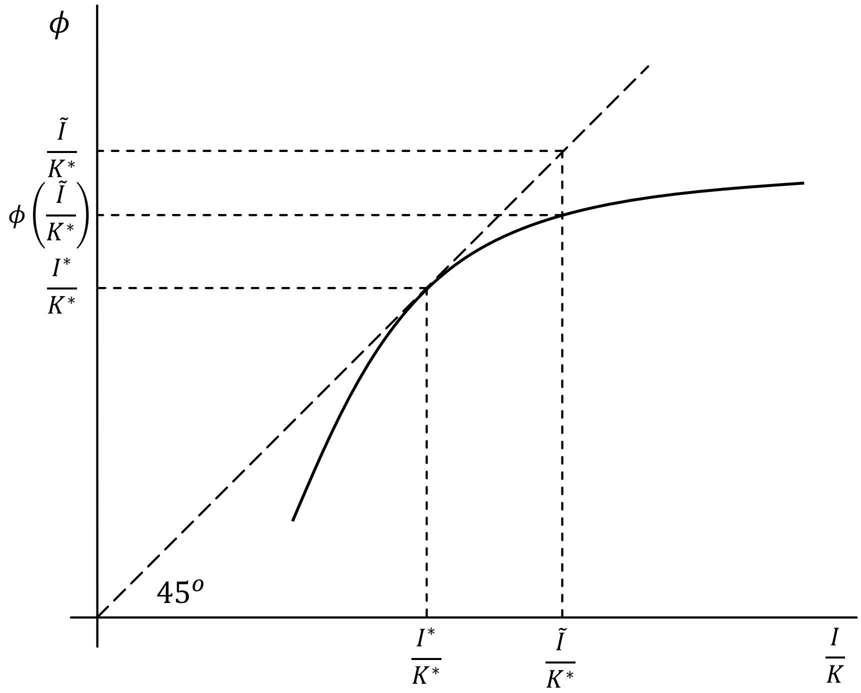

Fig. 2. The Cost of Adjusting Capital Stock

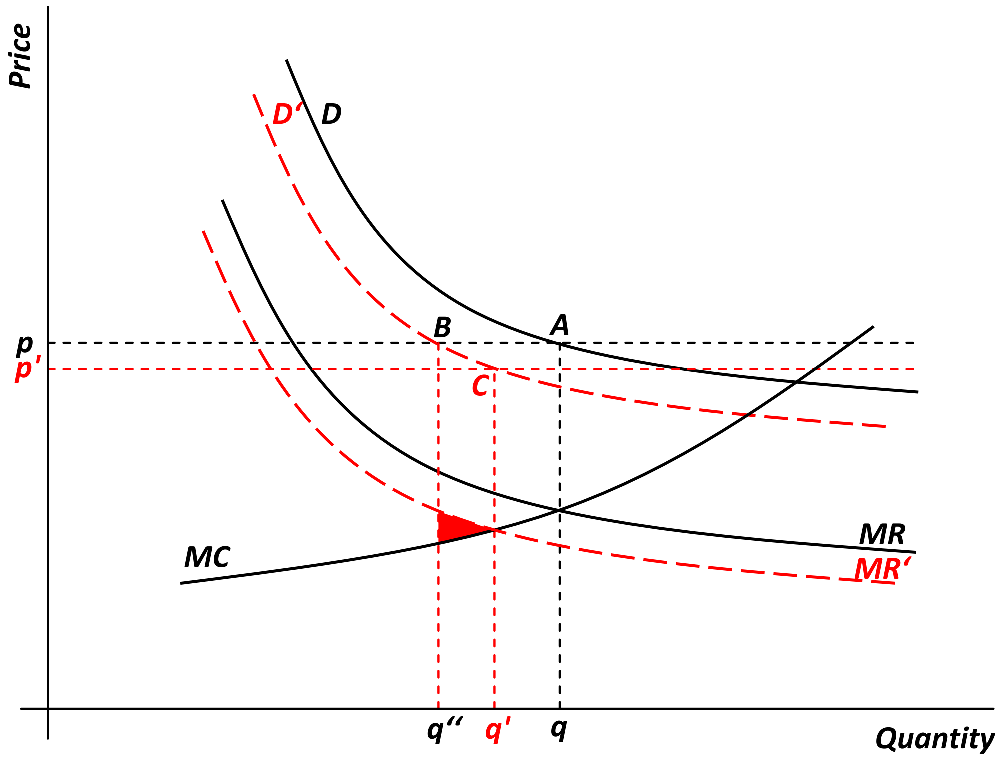

Fig. 3. The Incentive for Price Adjustment

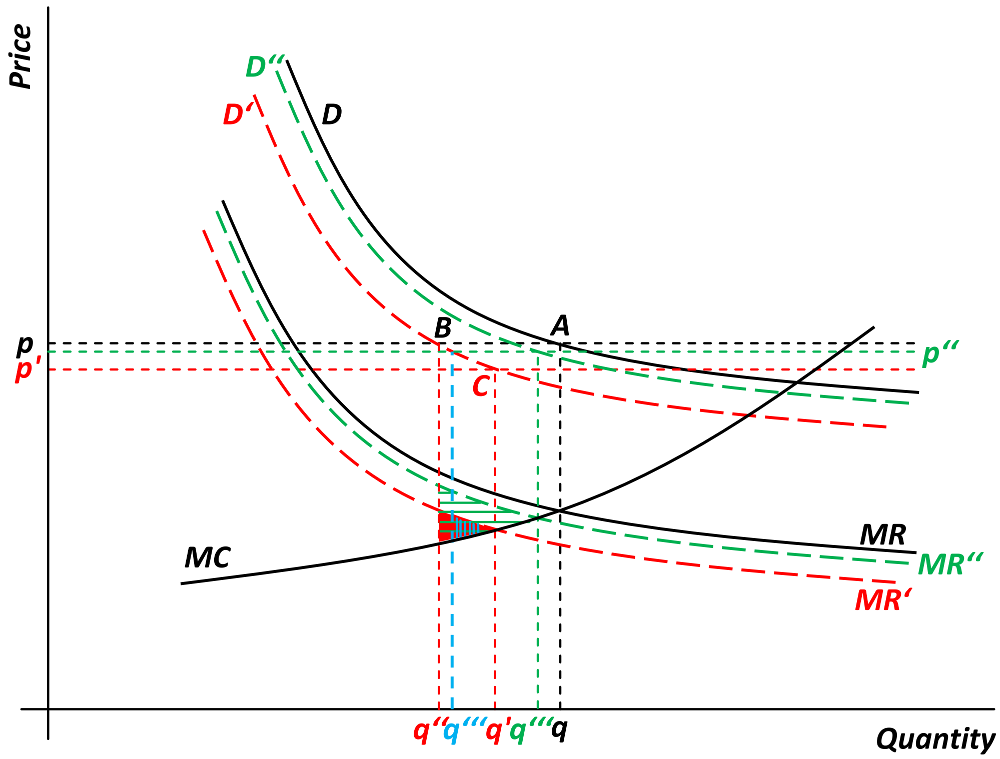

Fig. 4. Aggregate Demand Externality and the Coordination Failure