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From Real Business Cycle to New Keynesian Economics (II): General Equilibrium

People say that you are crazy. But are you crazy enough?

—Niels Bohr

1 Introduction

In this chapter we proceed with characterizing the general equilibrium, then we log-linearize the system and simulate the local dynamics.

Next, in order to address the central issues, we simplify the system into two equations. Then it becomes clear with the *new Keynesian perspective*, and how it differs from the old Keynesians. In the end, the (in-)determinancy problem with monetary rules is discussed, and the limitation of the popular two equation system is investigated.

2 The Equilibrium

In the very beginning let's make a brief conclusion of what we have already achieved in the last chapter. First, we have to aggregate all the individual optimality conditions into the conditions for the whole economy; Second, we characterize the general equilibrium of the economy and see how these equations intrinsically relate to each other.

2.1 Aggregation

In the last chapter we discussed the profit maximization problems for the three types of firms. However, what we got there was the efficiency conditions for individual firms. Now in order to relate these results to the macro indicators, we have to aggregate each variable that appears on the individual levels.

2.1.1 Aggregation of Factors

Remember that labor is only used as an input by the wholesale firms, so the aggregate labor input is

$$N_t = \int_0^1 N_t(z)dz.$$

And in the beginning of each period t the capital holdings from the last period, K_{t-1} is rented by the wholesale firms to produce the intermediate goods, therefore

$$K_{t-1} = \int_0^1 K_t(z)dz.$$

2.1.2 *Aggregation of Output*

The aggregate output in the economy is the total amount of the final goods, Y_t, which is produced by the final goods producers that assemble all the intermediate goods together

$$
\begin{aligned}
Y_t &= \left[\int_0^1 Y_t(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}} \\
&= \left\{ \int_0^1 \left[A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \right]^{\frac{\epsilon-1}{\epsilon}} dz \right\}^{\frac{\epsilon}{\epsilon-1}} .
\end{aligned}
$$

However, given that the wholesale firms adjust their prices in a staggering manner the firms' relative prices, $\frac{P_t(z)}{P_t}$, differ from each other, therefore the firms' output levels, $Y_t(z)$, are different — This makes the computation of Y_t very difficult.

Let's consider an alternative measure of aggregate output, Y_t^z, which is the aggregate output of the intermediate goods from the wholesale firms such that

$$
\begin{aligned}
Y_t^z &= \int_0^1 Y_t(z) dz \\
&= \int_0^1 \left[A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \right] dz.
\end{aligned}
$$

Then it turns out that the computation of Y_t^z is very simple: Remember that the wholesale firms' production fuction makes constant return to scale, then it doesn't matter whether we produce the intermediate goods in a continuum of firms or in a single wholesale firm which uses the aggregate labor N_t and aggregagte capital stock K_{t-1} as inputs, i.e.

$$
\begin{aligned}
Y_t^z &= \int_0^1 \left[A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \right] dz \\
&= A_t N_t^{\alpha} K_{t-1}^{1-\alpha}.
\end{aligned}
$$

Then the next step is to build a link between Y_t and Y_t^z. Remember that no matter a wholesale firm z is adjusting its price at period t or not, the demand for its product $Y_t(z)$ is solely determined by the aggregate output Y_t and the relative price $\frac{P_t(z)}{P_t}$

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t.$$

Apply this expression in the definition of Y_t^z

$$\begin{aligned}
Y_t^z &= \int_0^1 Y_t(z)dz \\
&= \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} Y_t dz \\
&= Y_t \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} dz, \\
A_t N_t^{\alpha} K_{t-1}^{1-\alpha} &= Y_t \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} dz.
\end{aligned}$$

Let's define a new variable to finish the aggregation of production

$$s_t = \int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{-\epsilon} dz.$$

Note that in any period t there are a share $1-\theta$ of the firms adjusting their prices following the optimal strategy P_t^* and a share θ of the firms which do not adjust their prices, the multiplier s_t between Y_t^z and Y_t can be furtherly expressed as

$$\begin{aligned}
s_t &= \int_0^{1-\theta} \left[\frac{P_t^*(z)}{P_t}\right]^{-\epsilon} dz + \int_{1-\theta}^1 \left[\frac{P_{t-1}(z)}{P_t}\right]^{-\epsilon} dz \\
&= (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} \int_{1-\theta}^1 \left[\frac{P_{t-1}(z)}{P_{t-1}}\right]^{-\epsilon} dz \\
&= (1-\theta)\tilde{P}_t^{*-\epsilon} + \theta(1+\pi_t)^{\epsilon} \int_0^1 \left[\frac{P_{t-1}(z)}{P_{t-1}}\right]^{-\epsilon} dz \\
&= (1-\theta)\tilde{P}_t^{*-\epsilon} + \theta(1+\pi_t)^{\epsilon} s_{t-1}.
\end{aligned}$$

Now it's shown that s_t can be written in a recursive manner with an aggregate variable $\tilde{P}_t^* = \frac{P_t^*}{P_t}$, i.e. the relative price between the optimally adjusted price in period t and current price level, and the aggregate output can be expressed as

$$Y_t^z = A_t N_t^{\alpha} K_{t-1}^{1-\alpha} = s_t Y_t.$$

Since s_t measures the gap between aggregate output of intermediate goods and final goods, it would be interesting to find out how big it is. To see this, define $\zeta_t = \left[\frac{P_t(z)}{P_t}\right]^{1-\epsilon}$, and obviously $s_t = \int_0^1 \zeta_t^{\frac{\epsilon}{\epsilon-1}} dz$. Notice that

$$\begin{aligned}
\left(\int_0^1 \zeta_t dz\right)^{\frac{\epsilon}{\epsilon-1}} &= \left\{\int_0^1 \left[\frac{P_t(z)}{P_t}\right]^{1-\epsilon} dz\right\}^{\frac{\epsilon}{\epsilon-1}} \\
&= P_t^{\epsilon}\left\{\left[\int_0^1 P_t(z)^{1-\epsilon} dz\right]^{\frac{1}{1-\epsilon}}\right\}^{-\epsilon} \\
&= 1
\end{aligned}$$

using the definition of price index for the last step, one can see that

$$1 = \left(\int_0^1 \zeta_t dz\right)^{\frac{\epsilon}{\epsilon-1}} \leq \int_0^1 \zeta_t^{\frac{\epsilon}{\epsilon-1}} dz = s_t$$

by Jensen's inequality because $\epsilon > 1$ and $\frac{\epsilon}{\epsilon-1} > 1$, and the equality holds only for ζ_t being constant, i.e. $P_t(z) = P_t$, $\forall z \in [0, 1]$ — when there exists no nominal rigidity in price adjustment. Then go back to the previous aggregated equation

$$Y_t^z = A_t N_t^{\alpha} K_{t-1}^{1-\alpha} = s_t Y_t.$$

Since s_t is bounded below by 1, Y_t is maximized when $s_t = 1$, i.e. $P_t(z)$ being the same for all the wholesale firms. This implies that given technology, labor input and capital accumulation (therefore aggregate output of intermediate goods) constant, the aggregate output is maximized only if there is no rigidity in price adjustment; and s_t measures the cost induced by the inefficient price dispersion out of the staggering price settings.

2.1.3 Aggregation of the Ratios

The labor demand of a wholesale firm is given by

$$\alpha Y_t(z) = \frac{1}{MC_t}\frac{W_t}{P_t} N_t(z).$$

This implies that $\forall z \in [0, 1]$

$$\frac{N_t(z)}{Y_t(z)} = MC_t \frac{\alpha P_t}{W_t},$$

in which the right hand side is only determined by the aggregate variables and remains the same across all the firms [1] . Using the knowledge from school mathematics that

$$\begin{aligned} &\frac{x_1}{y_1} = \frac{x_2}{y_2} = \ldots = \frac{x_n}{y_n} = \text{constant } \alpha \\ \Rightarrow &\frac{\sum_{i=1}^n x_i}{\sum_{i=1}^n y_i} = \alpha, \text{ with } \sum_{i=1}^n y_i \neq 0 \end{aligned}$$

we can immediately see that

$$\frac{N_t}{Y_t^z} = \frac{\int_0^1 N_t(z)dz}{\int_0^1 Y_t(z)dz} = \frac{N_t(z)}{Y_t(z)} = MC_t \frac{\alpha P_t}{W_t}. \tag{1}$$

Apply the same approach to the capital input of a wholesale firm

$$(1-\alpha)Y_t(z) = \frac{1}{MC_t} Z_t K_t(z),$$

one can see that the aggregated version of it turns out to be

$$\frac{K_{t-1}}{Y_t^z} = \frac{\int_0^1 K_t(z)dz}{\int_0^1 Y_t(z)dz} = \frac{K_t(z)}{Y_t(z)} = (1-\alpha) MC_t \frac{1}{Z_t}. \tag{2}$$

From the optimality condition for input of a capital producer

$$Q_t \phi' \left(\frac{I_t(j)}{K_t(j)} \right) = 1,$$

one can see that

$$\frac{I_t(j)}{K_t(j)} = (\phi')^{-1} \left(\frac{1}{Q_t} \right)$$

[1] For models with firm-specific capital, e.g. Sveen and Weinke (2005, 2007), Altig, Christiano, Eichenbaum and Linde (2005) and so on, it is no longer true that all firms face identical marginal costs, and the aggregation depends on how the firms are differentiated.

and again the ratio is constant across (or exogenous for) all the firms. Therefore by aggregation

$$\frac{I_t}{K_{t-1}} = \frac{\int_0^1 I_t(j)dj}{\int_0^1 K_t(j)dj} = \frac{I_t(j)}{K_t(j)} = (\phi')^{-1}\left(\frac{1}{Q_t}\right), \tag{3}$$

$$Q_t\phi'\left(\frac{I_t}{K_{t-1}}\right) = 1. \tag{4}$$

2.2 *General Equilibrium*

After aggregation we are able to characterize the equilibrium for the whole economy. Following Debreu (1959) the general equilibrium in this economy is a feasible plan

$$\{C_t, Y_t, I_t, N_t, K_t, M_t, B_t, TR_t, G_t, Z_t, W_t, P_t, R_t^n, Q_t, MC_t\}_{t=0}^{+\infty}$$

which satisfies the following conditions

- The representative consumer maximizes her utility;
- All of the firms maximize their profits;
- Market clearing, such that for each commodity its aggregate supply equals to its aggregate demand.

Now we can put all the corresponding conditions together, which we have already obtained so far. The equations are listed in the following *blocks* (as in Woodford, 2003) by their intrinsic relations, and the variables on the individual levels are already replaced by the aggregated ones.

IS Block

Part of the equations can be assembled in the *IS block*, by which we are able to pin down the traditional *IS curve* that determines the level of real aggregate demand associated with a given real interest rate — since the output is driven by the aggregate demand in our model, the IS block actually determines the equilibrium level of output under a given real interest rate.

$$Y_t = C_t + I_t + G_t, \tag{5}$$

$$Y_t = s_t Y_t^z, \tag{6}$$

$$s_t = (1-\theta)\tilde{P}_t^{*-\epsilon} + \theta(1+\pi_t)^{\epsilon} s_{t-1}, \tag{7}$$

$$\tilde{P}_t^* = \frac{P_t^*}{P_t}, \tag{8}$$

$$1 + \pi_t = \frac{P_t}{P_{t-1}}, \tag{9}$$

$$\frac{M_t}{P_t} = \left(\frac{1}{a_m}\right)^{-\frac{1}{\gamma m}} \left(1 - \frac{1}{R_t^n}\right)^{-\frac{1}{\gamma m}} C_t^{\frac{\gamma}{\gamma m}}, \tag{10}$$

$$1 = E_t\left[R_t\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right], \tag{11}$$

$$1 = E_t\left[\frac{Z_{t+1} + Q_{t+1}(1-\delta)}{Q_t}\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right], \tag{12}$$

$$(1-\alpha)Y_t^z = \frac{1}{MC_t}Z_tK_{t-1}, \tag{13}$$

$$1 = Q_t\phi'\left(\frac{I_t}{K_{t-1}}\right); \tag{14}$$

The central equation in this block is the Euler equation (11), which captures the representative agent's optimal intertemporal consumption decision, linking the private expenditure, i.e. part of the real demand of the economy, and the real interest rate. In times of shocks, such linkage bridges the monetary policy and real demand, and the shocks propagate in the economy via prices such as Z and Q, changing the equilibrium of the capital market, which generates further feedback to the goods market, making the original shocks persist.

AS (Aggregate Supply) Block

As another side of the coin, equations in *AS block* allow us to solve for the path of inflation with the paths of real output as well as the capital stock being given. Note that the capital stock is determined in the IS block, and here it is taken as an input to the AS block.

$$Y_t^z = A_tN_t^{\alpha}K_{t-1}^{1-\alpha}, \tag{15}$$

$$\alpha Y_t^z = \frac{1}{MC_t}\frac{W_t}{P_t}N_t, \tag{16}$$

$$\frac{W_t}{P_t}C_t^{-\gamma} = a_nN_t^{\gamma_n}, \tag{17}$$

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)P_t^{*1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{18}$$

$$P_t^* = (1+\mu)\sum_{i=0}^{+\infty}\psi_{t+i}MC_{t+i}^n, \tag{19}$$

$$\psi_{t+i} = \frac{E_t\left[\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}, \tag{20}$$

$$MC_t^n = P_t MC_t; \tag{21}$$

Capital Accumulation

The capital is accumulated as a balance between the gain from new capital purchase and the loss from depreciations.

$$K_t = \phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} + (1-\delta)K_{t-1}; \tag{22}$$

The Rules Block

The government may intervene the economy via fiscal and monetary policies, as two examples in the following.

— The Fiscal Rule

$$\frac{M_t - M_{t-1}}{P_t} = G_t + TR_t; \tag{23}$$

— The Monetary Rule

$$R_t^n = R^{n*}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y^*}\right)^{\gamma_y} e^{\epsilon_t^r}. \tag{24}$$

3 The Log-Linearized System

Many of the equations characterizing general equilibrium are non-linear, and this brings much difficulties in computations. Therefore it makes sense for us to concentrate on the local behavior around the steady state, and log-linearization with first order Taylor expansion would significantly simplify the system, making it easier to build some intuitions behind the equations.

3.1 Log-Linearizing the IS Block

Equations (5) — (14) are to be log-linearized in this section.

3.1.1 The Resource Constraint of the Economy

First divide the both sides of the economy's resource constraint

$$Y_t = C_t + I_t + G_t$$

by Y_t, and execute the first-order Taylor expansion

$$\begin{aligned} 1 &= \frac{C_t}{Y_t} + \frac{I_t}{Y_t} + \frac{G_t}{Y_t} \\ &= \frac{C^*}{Y^*}\left(\hat{c}_t - \hat{y}_t\right) + \frac{I^*}{Y^*}\left(\hat{i}_t - \hat{y}_t\right) + \frac{G^*}{Y^*}\left(\hat{g}_t - \hat{y}_t\right), \end{aligned}$$

merge all the $\hat{y}_t$ terms and get

$$\hat{y}_t = \frac{C^*}{Y^*}\hat{c}_t + \frac{I^*}{Y^*}\hat{i}_t + \frac{G^*}{Y^*}\hat{g}_t. \tag{25}$$

3.1.2 The Money Demand

Equation (10) characterizes the money demand. Take logs on both sides and linearize it,

$$\begin{aligned} \ln\left(\frac{M_t}{P_t}\right) &= \ln\left[\left(\frac{1}{a_m}\right)^{-\frac{1}{\gamma_m}}\left(1 - \frac{1}{R_t^n}\right)^{-\frac{1}{\gamma_m}} C_t^{\frac{\gamma}{\gamma_m}}\right], \\ \ln\left(\frac{M_t}{M^*}\right) - \ln\left(\frac{P_t}{P^*}\right) &= -\frac{1}{\gamma_m}\ln\left(\frac{r_t^n}{r^{n*}}\right) + \frac{\gamma}{\gamma_m}\ln\left(\frac{C_t}{C^*}\right), \end{aligned}$$

then replace the percentage changes with hat variables,

$$\hat{m}_t - \hat{p}_t = \frac{\gamma}{\gamma_m}\hat{c}_t - \frac{1}{\gamma_m}\hat{r}_t^n. \tag{26}$$

3.1.3 The Euler Equation

The Euler equation characterizes the agent's intertemporal consumption decisions. Take logs of both sides and then implement the first-order Taylor expansion

$$1 = E_t\left[R_t\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right],$$

$$
\begin{aligned}
1 &= E_t\left[R_t^n\left(\frac{P_t}{P_{t+1}}\right)\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right],\\
0 &= \ln\left\{E_t\left[R_t^n\left(\frac{P_t}{P_{t+1}}\right)\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]\right\},\\
&= E_t\left\{\ln\left[R_t^n\left(\frac{P_t}{P_{t+1}}\right)\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]\right\} + \frac{1}{2}\text{var}_t\left\{\ln\left[R_t^n\left(\frac{P_t}{P_{t+1}}\right)\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]\right\}
\end{aligned}
$$

— Here we use the fact that $\xi_t = R_t^n\left(\frac{P_t}{P_{t+1}}\right)\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ being log-normal implies that $E_t[\xi_t] = \exp\left(E_t[\ln\xi_t] + \frac{1}{2}\text{var}[\ln\xi_t]\right)$. Continue with taking logs and implement the first-order Taylor expansion

$$
\begin{aligned}
0 &= E_t\left\{\ln\left[R^{n*}\left(1+\frac{R_t^n-R^{n*}}{R^{n*}}\right)\left(\frac{1}{1+\frac{P_{t+1}-P_t}{P_t}}\right)\beta\left(1+\frac{C_{t+1}-C_t}{C_t}\right)^{-\gamma}\right]\right\}\\
&\quad+\frac{1}{2}\text{var}_t\left\{\ln\left[R^{n*}\left(1+\frac{R_t^n-R^{n*}}{R^{n*}}\right)\left(\frac{1}{1+\frac{P_{t+1}-P_t}{P_t}}\right)\beta\left(1+\frac{C_{t+1}-C_t}{C_t}\right)^{-\gamma}\right]\right\}\\
&\approx E_t\left\{\ln\left[R^{n*}(1+\hat{r}_t^n)\left(\frac{1}{1+\hat{p}_{t+1}-\hat{p}_t}\right)\beta(1+\hat{c}_{t+1}-\hat{c}_t)^{-\gamma}\right]\right\}\\
&\quad+\frac{1}{2}\text{var}_t\left\{\ln\left[R^{n*}(1+\hat{r}_t^n)\left(\frac{1}{1+\hat{p}_{t+1}-\hat{p}_t}\right)\beta(1+\hat{c}_{t+1}-\hat{c}_t)^{-\gamma}\right]\right\}\\
&\approx E_t\left[\ln R^{n*}+\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)+\ln\beta-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right]\\
&\quad+\frac{1}{2}\text{var}_t\left[\ln R^{n*}+\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)+\ln\beta-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right],
\end{aligned}
$$

then pass the expectation and variance operators through and get

$$
\begin{aligned}
0 &= \ln R^{n*}+\ln\beta+E_t\left[\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right]\\
&\quad+\frac{1}{2}\text{var}_t\left[\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right].
\end{aligned}
\tag{27}
$$

Estimate equation (27) in the steady state in which

$$
E_t^*\left[\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right]=0,
$$

implying that the following equation holds all the time

$$
\ln R^{n*}+\ln\beta+\frac{1}{2}\text{var}_t\left[\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right]=0,
$$

but this just means that equation (27) can be simplified as

$$
0 = E_t\left[\hat{r}_t^n-(\hat{p}_{t+1}-\hat{p}_t)-\gamma(\hat{c}_{t+1}-\hat{c}_t)\right]. \tag{28}
$$

Then continue to solve for $\hat{c}_t$

$$\hat{c}_t = -\frac{1}{\gamma}\left[\hat{r}^n_t - E_t\left(\hat{p}_{t+1} - \hat{p}_t\right)\right] + E_t\hat{c}_{t+1}, \tag{29}$$

$$\hat{c}_t = -\sigma\left[\hat{r}^n_t - E_t\left(\hat{p}_{t+1} - \hat{p}_t\right)\right] + E_t\hat{c}_{t+1} \tag{30}$$

in which $\sigma = \frac{1}{\gamma}$ is the instantaneous elasticity of substitutions. Using the fact that $E_t\pi_{t+1} = E_t\left(\hat{p}_{t+1} - \hat{p}_t\right)$, equation (30) can be simplified as

$$\hat{c}_t = -\sigma\left[\hat{r}^n_t - E_t\pi_{t+1}\right] + E_t\hat{c}_{t+1} \tag{31}$$

3.1.4 The Capital Return

Combine equations (6), (12) and (13) and get

$$E_t\left\{\left[(1-\alpha)MC_{t+1}\frac{s_{t+1}Y_{t+1}}{K_t} + Q_{t+1}(1-\delta)\right]\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right\} = E_t\left[Q_tR_t\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right],$$

$$\begin{aligned}&E_t\left[(1-\alpha)MC^*\frac{s^*Y^*}{K^*}\left(1 + \hat{s}_{t+1} + \hat{mc}_{t+1} + \hat{y}_{t+1} - \hat{k}_t\right) + Q^*(1-\delta)\left(1 + \hat{q}_{t+1}\right)\right]\\&= E_t\left[Q^*R^{n*}\left(1 + \hat{q}_t + \hat{r}^n_t + \hat{p}_{t+1} - \hat{p}_t\right)\right].\end{aligned}$$

Note that in the steady state,

$$Q^* = 1,$$

and all the hat-terms are zero, therefore we get the relation between all the steady-state values

$$(1-\alpha)MC^*\frac{s^*Y^*}{K^*} + Q^*(1-\delta) = Q^*R^{n*}. \tag{32}$$

Use equation (32) to eliminate the redundant terms, and we get

$$\begin{aligned}&E_t\left[(1-\alpha)MC^*\frac{s^*Y^*}{K^*}\left(\hat{s}_{t+1} + \hat{mc}_{t+1} + \hat{y}_{t+1} - \hat{k}_t\right) + (1-\delta)\hat{q}_{t+1}\right]\\&= E_t\left[R^{n*}\left(\hat{q}_t + \hat{r}^n_t + \hat{p}_{t+1} - \hat{p}_t\right)\right].\end{aligned}$$

Define a constant v as

$$v = \frac{1-\delta}{(1-\alpha)MC^* \frac{s^*Y^*}{K^*} + (1-\delta)},$$

the linearized equation can be simplified as

$$E_t\left[(1-v)\left(\hat{s}_{t+1} + \hat{mc}_{t+1} + \hat{y}_{t+1} - \hat{k}_t\right) + v\hat{q}_{t+1} - \hat{q}_t\right] = E_t\left(\hat{r}_t^n + \hat{p}_{t+1} - \hat{p}_t\right). \tag{33}$$

3.1.5 Demand for Investment Goods

Equation (14) characterizes the demand for investment goods. Implement the first-order Taylor expansion around the steady state,

$$Q_t \phi'\left(\frac{I_t}{K_{t-1}}\right) = 1,$$

$$Q^* \phi'\left(\frac{I^*}{K^*}\right) + \phi'\left(\frac{I^*}{K^*}\right) Q^* \hat{q}_t + Q^* \phi''\left(\frac{I^*}{K^*}\right)\frac{I^*}{K^*}\left(\hat{i}_t - \hat{k}_{t-1}\right) = 1.$$

Use the facts that $Q^*\phi'\left(\frac{I^*}{K^*}\right) = 1$, and solve for $\hat{q}_t$

$$\hat{q}_t = -\frac{\phi''\left(\frac{I^*}{K^*}\right)\frac{I^*}{K^*}}{\phi'\left(\frac{I^*}{K^*}\right)}\left(\hat{i}_t - \hat{k}_{t-1}\right), \tag{34}$$

$$\hat{q}_t = \eta\left(\hat{i}_t - \hat{k}_{t-1}\right) \tag{35}$$

in which η captures the curvature of $\phi(\cdot)$ around the steady state, i.e. the higher η is, the higher the investment cost is incurred when the system deviates from the steady state.

3.2 Log-Linearizing the AS Block

Equations (15) — (21) are to be log-linearized in this section.

3.2.1 The Aggregate Output

Start from the relation between the aggregate output and the factors input, apply log-linearization directly and get the linearized equations immediately

$$s_t Y_t = A_t N_t^{\alpha} K_{t-1}^{1-\alpha},$$

$$\frac{s_t Y_t}{s^* Y^*} = \frac{A_t N_t^{\alpha} K_{t-1}^{1-\alpha}}{A^* N^{*\alpha} K^{*1-\alpha}},$$

$$\ln\left(\frac{s_t}{s^*}\right)+\ln\left(\frac{Y_t}{Y^*}\right)=\ln\left(\frac{A_t}{A^*}\right)+\alpha\ln\left(\frac{N_t}{N^*}\right)+(1-\alpha)\ln\left(\frac{K_{t-1}}{K^*}\right).$$

Replace the ratios with hat variables and get

$$\hat{s}_t+\hat{y}_t=a_t+\alpha\hat{n}_t+(1-\alpha)\hat{k}_{t-1}. \tag{36}$$

3.2.2 The Labor Supply

The representative household's labor supply is governed by equation (17)

$$\frac{W_t}{P_t}C_t^{-\gamma}=a_nN_t^{\gamma_n},$$

combine with the wholesale firm's labor demand function, equation (16), and get

$$\alpha s_tY_t=\frac{1}{MC_t}C_t^{\gamma}a_nN_t^{\gamma_n}N_t.$$

Again directly apply the trick of log-linearization and get

$$\hat{s}_t+\hat{y}_t=-\hat{mc}_t+\gamma\hat{c}_t+(1+\gamma_n)\,\hat{n}_t, \tag{37}$$

$$\hat{s}_t+\hat{y}_t+\hat{mc}_t-\gamma\hat{c}_t=(1+\gamma_n)\,\hat{n}_t. \tag{38}$$

3.2.3 Optimal Price Adjustment (The New Keynesian Phillips Curve)

To find the linear form of price level P_t defined by equation (18), one has to solve for P_t^* first. Start from the expression for the optimally adjusted price P_t^*

$$P_t^*=(1+\mu)\frac{\sum_{i=0}^{+\infty}\left\{E_t\left[\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}MC_{t+i}^n\right]\right\}}{E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]}.$$

Put the two summation terms on both sides for computational simplicity,

$$P_t^*E_t\left[\sum_{i=0}^{+\infty}\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}\right]=(1+\mu)\sum_{i=0}^{+\infty}\left\{E_t\left[\frac{\theta^i}{R_{t,t+i}}\left(\frac{1}{P_{t+i}}\right)^{1-\epsilon}Y_{t+i}MC_{t+i}^n\right]\right\},$$

then divide both sides by P_t and replace the stochastic discount rate $R_{t,t+i}$ in term of consumption ratio

$$\frac{P_t^*}{P_t}E_t\left[\sum_{i=0}^{+\infty}(\theta\beta)^i\left(\frac{C_{t+i}}{C_t}\right)^{-\gamma}P_{t+i}^{\epsilon-1}Y_{t+i}\right]=(1+\mu)\sum_{i=0}^{+\infty}\left\{E_t\left[(\theta\beta)^i\left(\frac{C_{t+i}}{C_t}\right)^{-\gamma}P_{t+i}^{\epsilon-1}Y_{t+i}\frac{MC_{t+i}^n}{P_t}\right]\right\},$$

and log-linearize the equation

$$\begin{aligned}
&E_t\left(P^{*\epsilon-1}Y^*\left[\sum_{i=0}^{+\infty}(\theta\beta)^i\right](\hat{p}_t^*-\hat{p}_t)+P^{*\epsilon-1}Y^*\left\{\sum_{i=0}^{+\infty}(\theta\beta)^i\left[\hat{\lambda}_{t,t+i}+(\epsilon-1)\hat{p}_{t+i}+\hat{y}_{t+i}\right]\right\}\right)\\
&=E_t\left\{P^{*\epsilon-1}Y^*\sum_{i=0}^{+\infty}(\theta\beta)^i\left[\hat{\lambda}_{t,t+i}+(\epsilon-1)\hat{p}_{t+i}+\hat{y}_{t+i}+\hat{mc}_{t+i}^n-\hat{p}_t\right]\right\},
\end{aligned}$$

— Note that in the steady state $C_{t+i}=C_t=C^*$, then eliminate the redundant terms on both sides

$$\begin{aligned}
&\frac{1}{1-\theta\beta}(\hat{p}_t^*-\hat{p}_t)+E_t\left\{\sum_{i=0}^{+\infty}(\theta\beta)^i\left[\hat{\lambda}_{t,t+i}+(\epsilon-1)\hat{p}_{t+i}+\hat{y}_{t+i}\right]\right\}\\
&=E_t\left\{\sum_{i=0}^{+\infty}(\theta\beta)^i\left[\hat{\lambda}_{t,t+i}+(\epsilon-1)\hat{p}_{t+i}+\hat{y}_{t+i}+\hat{mc}_{t+i}^n\right]\right\}-\frac{1}{1-\theta\beta}\hat{p}_t.
\end{aligned}$$

Note that the two expectation terms are only different in the part of $\hat{mc}_{t+i}^n$, further arrange the equation to get

$$\hat{p}_t^*=(1-\theta\beta)E_t\left[\sum_{i=0}^{+\infty}(\theta\beta)^i\hat{mc}_{t+i}^n\right]. \tag{39}$$

Next we come back to the price index P_t

$$\begin{aligned}
P_t&=\left[\theta P_{t-1}^{1-\epsilon}+(1-\theta)P_t^{*1-\epsilon}\right]^{\frac{1}{1-\epsilon}},\\
P_t^{1-\epsilon}&=\theta P_{t-1}^{1-\epsilon}+(1-\theta)P_t^{*1-\epsilon},\\
P^{*1-\epsilon}+(1-\epsilon)P^{*1-\epsilon}\hat{p}_t&=\theta P^{*1-\epsilon}+(1-\theta)P^{*1-\epsilon}+(1-\epsilon)P^{*1-\epsilon}\left[\theta\hat{p}_{t-1}+(1-\theta)\hat{p}_t^*\right].
\end{aligned}$$

Eliminate the redundant terms and one can get

$$\hat{p}_t=\theta\hat{p}_{t-1}+(1-\theta)\hat{p}_t^*, \tag{40}$$
$$\hat{p}_t^*=\frac{\hat{p}_t-\theta\hat{p}_{t-1}}{1-\theta}. \tag{41}$$

Since the price index works for all t, then update equations (40) and (41) to get

$$E_t\hat{p}_{t+1} = E_t\left[\theta\hat{p}_t + (1-\theta)\hat{p}^*_{t+1}\right], \tag{42}$$
$$E_t\left(\hat{p}^*_{t+1}\right) = \frac{E_t\hat{p}_{t+1} - \theta\hat{p}_t}{1-\theta}. \tag{43}$$

Then we combine what we got regarding P_t and P^*_t. Continue with equation (39)

$$\begin{aligned}
\hat{p}^*_t &= (1-\theta\beta)\left\{E_t\left[\hat{mc}^n_t + \sum_{i=1}^{+\infty}(\theta\beta)^i\hat{mc}^n_{t+i}\right]\right\}\\
&= (1-\theta\beta)\left\{\hat{mc}^n_t + \theta\beta E_t\left[\sum_{i=0}^{+\infty}(\theta\beta)^i\hat{mc}^n_{t+1+i}\right]\right\}\\
&= (1-\theta\beta)\hat{mc}^n_t + \theta\beta E_t\left(\hat{p}^*_{t+1}\right)\\
&= (1-\theta\beta)\left(\hat{mc}_t + \hat{p}_t\right) + \theta\beta\frac{E_t\hat{p}_{t+1} - \theta\hat{p}_t}{1-\theta}
\end{aligned}$$

to express $\hat{p}_t$ in a recursive way. $\hat{p}^*_t$ is then replaced by $\hat{p}_t$ via inserting equation (41)

$$\begin{aligned}
\frac{\hat{p}_t - \theta\hat{p}_{t-1}}{1-\theta} &= (1-\theta\beta)\left(\hat{mc}_t + \hat{p}_t\right) + \theta\beta\frac{E_t\hat{p}_{t+1} - \theta\hat{p}_t}{1-\theta},\\
\hat{p}_t - \theta\hat{p}_{t-1} &= (1-\theta\beta)(1-\theta)\left(\hat{mc}_t + \hat{p}_t\right) + \theta\beta\left(E_t\hat{p}_{t+1} - \theta\hat{p}_t\right),\\
\theta\hat{p}_t - \theta\hat{p}_{t-1} &= (1-\theta\beta)(1-\theta)\hat{mc}_t + \theta\beta E_t\hat{p}_{t+1} - \theta\beta\hat{p}_t,\\
\theta\pi_t &= (1-\theta\beta)(1-\theta)\hat{mc}_t + \theta\beta E_t\pi_{t+1}
\end{aligned}$$

— The difference, $\hat{p}_t - \hat{p}_{t-1}$, is simply the inflation rate π_t for period t. Define a constant

$$\kappa = \frac{(1-\theta\beta)(1-\theta)}{\theta},$$

and the equation above can be simplified as

$$\pi_t = \kappa\hat{mc}_t + \beta E_t\pi_{t+1}, \tag{44}$$

which is often called *new Keynesian Phillips curve*.

3.3 Log-Linearizing the Capital Accumulation

In each period t, the last period capital stock K_{t-1} is carried over after being depreciated at a rate of δ, and the investment I_t is added into the capital stock via the capital production procedure,

$$K_t = \phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} + (1-\delta)K_{t-1}. \tag{45}$$

Divide both sides by K_{t-1} and implement the first-order Taylor expansion

$$\begin{aligned}
\frac{K_t}{K_{t-1}} &= \phi\left(\frac{I_t}{K_{t-1}}\right) + (1-\delta),\\
1 + \hat{k}_t - \hat{k}_{t-1} &= \phi\left(\frac{I}{K}\right) + \phi'\left(\frac{I}{K}\right)\frac{I}{K}\hat{i}_t - \phi'\left(\frac{I}{K}\right)\frac{I}{K}\hat{k}_{t-1} + (1-\delta),
\end{aligned}$$

Note that in the steady state, equation (45) becomes

$$\begin{aligned}
K^* &= \phi\left(\frac{I^*}{K^*}\right)K^* + (1-\delta)K^*,\\
\phi\left(\frac{I^*}{K^*}\right) &= \frac{I^*}{K^*}\\
&= \delta.
\end{aligned}$$

Then the linearized equation turns out to be

$$\hat{k}_t = \delta\hat{\hat{i}}_t + (1-\delta)\hat{k}_{t-1}. \tag{46}$$

3.4 Log-Linearizing the Rules

In the end we log-linearize the rules.

3.4.1 The Fiscal Rule

For the example of fiscal rule in our model

$$\frac{M_t - M_{t-1}}{P_t} = \Delta m_t = G_t + TR_t,$$

the log-linearized form turns out to be

$$\Delta m^*\Delta\hat{m}_t = G^*\hat{g}_t + TR^*\hat{TR}_t. \tag{47}$$

3.4.2 The Monetary Rule

For the example of monetary rule

$$R_t^n = R^{n*}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y^*}\right)^{\gamma_y} e^{\epsilon_t^r}$$

in which central bank responds to inflation and output gap, the log-linearized form turns out to be

$$\hat{r}_t^n = \gamma_\pi \pi_t + \gamma_y \left(\hat{y}_t - \hat{y}_t^*\right) + \epsilon_t^r. \tag{48}$$

3.5 *On the Linearized General Equilibrium: The "Micro-Foundations"*

Now all the general equilibrium conditions have been log-linearized. Remember that traditional $IS - LM$ paradigm works well for analyzing macro issues, so it would be good to relate these equations to the paradigm that we are familiar with. For long time traditional $IS - LM$ paradigm is criticized for "starting from the curves", lacking for a sound, convincing foundation behind the graphs. But now in our model, we have already shown that all the equations are derived from the optimizing behavior of all the agents in the economy; therefore we can link these equations with the curves, providing the sound "micro-foundations"for the traditional paradigm.

IS Curve

IS curve shows the equilibrium in the goods market, which is captured in equation (31)

$$\hat{c}_t = -\frac{1}{\gamma}\left[\hat{r}_t^n - E_t\pi_{t+1}\right] + E_t\hat{c}_{t+1}. \tag{49}$$

One can solve equation (49) recursively and this yields

$$\hat{c}_t = -\frac{1}{\gamma}E_t\left[\sum_{i=0}^{+\infty}\left(\hat{r}_{t+i}^n - \pi_{t+1+i}\right)\right]. \tag{50}$$

Note that $r_t^n - E_t\pi_{t+1}$ defines the real interest rate, i.e. the level of one-period yield of the bonds, in each period and $\hat{r}_t^n - E_t\pi_{t+1}$ is just the measure of the deviation from its steady state level (with zero inflation). Therefore equation (50) implies that the percentage deviation in current period consumption is proportional to the sum of current and anticipated deviations in the return of bonds.

LM Curve

LM curve shows the equilibrium in the money market, which is captured in equation (26)

$$\hat{m}_t - \hat{p}_t = \frac{\gamma}{\gamma_m}\hat{c}_t - \frac{1}{\gamma_m}\hat{r}_t^n. \tag{51}$$

AS Curve

AS curve reflects the firms' incentive for production at some given factor prices, which is captured in equations (36) and (38). Combine these equations to eliminate $\hat{s}_t$ and get a reduced form

$$\hat{s}_t + \hat{y}_t + \hat{mc}_t - \gamma\hat{c}_t - \hat{s}_t - \hat{y}_t = (1 + \gamma_n)\,\hat{n}_t - a_t - \alpha\hat{n}_t - (1-\alpha)\hat{k}_{t-1},$$

rearrange to get the expression for $\hat{mc}_t$

$$\hat{mc}_t = \gamma\hat{c}_t + (1 - \alpha + \gamma_n)\,\hat{n}_t - a_t - (1-\alpha)\hat{k}_{t-1}. \tag{52}$$

The New Keynesian Phillips Curve

The basic lesson that the new Keynesian Phillips curve (44) tells us is that the inflation process is forward-looking, i.e. current inflation is a function of expected future inflation — At the time when a firm adjusts its price, it must take the future inflation into account because it is unable to adjust its price for a couple of periods in the future.

If we solve the new Keynesian Phillips curve (44) recursively for π_t, we can see that

$$\begin{aligned}\pi_t &= \kappa\hat{mc}_t + \beta E_t\pi_{t+1}\\ &= \kappa\sum_{i=0}^{+\infty}\beta^i\hat{mc}_{t+i}\end{aligned}$$

implying that inflation depends on current and expected marginal costs, which makes it different from the traditional Phillips curve.

Insert (52) to replace $\hat{mc}_t$ in equation (44)

$$\pi_t = \kappa\left[\gamma\hat{c}_t + (1 - \alpha + \gamma_n)\,\hat{n}_t - a_t - (1-\alpha)\hat{k}_{t-1}\right] + \beta E_t\pi_{t+1}, \tag{53}$$

implying that the change in real marginal cost $\hat{mc}_t$ also captures the excess demand $\hat{c}_t$. It follows that by equation (53) a higher demand causes scarcity of inputs and running capital at high intensity, which in turn results in high marginal costs in the future. So current inflation becomes higher.

3.6 Numerical Simulation

In our model the shocks are transmitted in various channels, therefore it's hard to see it clearly how the economy evolves upon shocks (which can be productivitiy shock, A_t as in real business cycle models, or monetary shock ϵ_t^r in the monetary rule) and numerical simulation is the only way to visualize the evolution. However, for lack of approapriate data, I am not able to produce proper graphs here. Figure 1 is taken from a similar study by Woodford (Woodford (2003), in which the capital is firm-specific), and the pattern for ours sould not be too different.

In the simulated model, there exists neither fiscal rule nor government expenditure, i.e. $G_t = 0$. The central bank intervenes the economy through the monetary rule (48), in which $\gamma_\pi = 2$ and $\gamma_y = 1$. Figure 1 shows how the economy evolves upon a shock of an unexpected monetary tightening (the nominal interest rate is increased by 1 percent): the curves in solid lines describe the case as in our model, such that the capital stock is adjusted by the capital accumulation procedure; and the curves in dotted lines describe the reference case as in the seminal Calvo-Yun model, such that the capital stock is constant (and with no depreciation).In order to make a better comparison, the models are calibrated in the way to produce the similar responses in output. Both models predict the same patterns of evolution: output, real marginal cost and inflation drop in the period of tightening, and the nominal interest rate is immediately adjusted downward by the monetary rule. Then the economy recovers as the time going on.

However, in our model with capital accumulation, the households are able to adjust their capital stock when the captial price and rental rate change. This makes the recovery process more persistent in the models with capital accumulation, which seems correct in the evolution of real interest rate.

4 Discussions

Although the system has been much simplified via log-linearization, it is still too complicated to see how everything works because there are too many transmission channels in our model.

So in order to uncover the driving forces behind the curtain, we will make more simplifications in the first place; then we'll see that the economic dynamics can be captured in two kernel equations and the *new Keynesian perspective* will be crystal clear. In the end we close the model by adding the role of monetary policy, and discuss which kind of monetary rule is desired.

4.1 Simplification

Suppose that there is neither capital accumulation nor shock in government expenditure, i.e. $\delta = 0$, $I = 0$, $\hat{g}_t = 0$, and the production function of the wholesale firms only involves labor input, i.e. $\alpha = 1$, $Y_t(z) = A_t N_t(z)$ (and $Y_t^z = A_t N_t$ by aggregation). Then we can quickly see that [2]

$$
\begin{aligned}
\hat{c}_t &= \hat{y}_t, && (54)\\
\hat{y}_t &= a_t + \hat{n}_t. && (55)
\end{aligned}
$$

---

[2] Precisely equation (55) should be

$$\hat{y}_t = a_t + \hat{n}_t - \hat{s}_t,$$

because $Y_t^z = \hat{s}_t Y_t$ in which $\hat{s}_t$ is the measure for price dispersion, as in Section 2.1.2. However, it can be shown (if I could finish the next chapter, I would show this as an example of *second order approximation*, :-P) that $\hat{s}_t$ is nearly constant if the system is stationary around the steady state. Therefore $\hat{s}_t$ can be absorbed in the exogenous term a_t.

However, without making use of second order approximation, there is an indirect way to see that it doesn't matter to drop off s_t here. Remember that we can define s_t from equations (7) and (8)

$$s_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} + \theta\left(\frac{P_t}{P_{t-1}}\right)^{\epsilon} s_{t-1},$$

let's have a look at how if we log-linearize it around the steady state.

$$
\begin{aligned}
s^* + s^*\hat{s}_t = (1-\theta)\left(\frac{P^*}{P^*}\right)^{-\epsilon} + \theta\left(\frac{P^*}{P^*}\right)^{\epsilon} s^* + (1-\theta)(-\epsilon)\Bigg[&\left(\frac{P^*}{P^*}\right)^{*-\epsilon-1}\frac{P^*}{P^*}\hat{p}_t^*\\
-\left(\frac{P^*}{P^*}\right)^{*-\epsilon-1}\frac{P^{*2}}{P^{*2}}\hat{p}_t\Bigg] &+ \epsilon\theta\left(\frac{P^*}{P^*}\right)^{\epsilon} s^*\left[\hat{p}_t - \hat{p}_{t-1}\right] + \theta\left(\frac{P^*}{P^*}\right)^{\epsilon} s^*\hat{s}_{t-1},\\
\hat{s}_t = (1-\theta)(-\epsilon)\left[\hat{p}_t^* - \hat{p}_t\right] + \epsilon\theta\pi_t + \theta\hat{s}_{t-1}.&
\end{aligned}
$$

Now apply what we got in equation (41)

$$\hat{p}_t^* = \frac{\hat{p}_t - \theta\hat{p}_{t-1}}{1-\theta}$$

to replace $\hat{p}_t^*$ in the expression for $\hat{s}_t$

$$
\begin{aligned}
\hat{s}_t &= (1-\theta)(-\epsilon)\left[\frac{\hat{p}_t - \theta\hat{p}_{t-1}}{1-\theta} - \hat{p}_t\right] + \epsilon\theta\pi_t + \theta\hat{s}_{t-1}\\
&= -\epsilon\left[\hat{p}_t - \theta\hat{p}_{t-1} - \hat{p}_t + \theta\hat{p}_t\right] + \epsilon\theta\pi_t + \theta\hat{s}_{t-1}\\
&= \theta\hat{s}_{t-1}.
\end{aligned}
$$

This means that in a steady state such that $P_t^* = P_t = P_{t-1} = P^*$, i.e. $\pi^* = 0$, up to the first order $\hat{s}_t$ can be written as

$$\hat{s}_t = \theta\hat{s}_{t-1},$$

which is a univariate autoregressive process and doesn't have any *real* consequences, so that we can ignore $\hat{s}_t$ in this case.

The new Keynesian Phillips curve (53) becomes

$$\pi_t = \kappa (\gamma + \gamma_n) \left(\hat{c}_t - \frac{1 + \gamma_n}{\gamma + \gamma_n} a_t \right) + \beta E_t \pi_{t+1} \tag{56}$$

in which κ remains the same

$$\kappa = \frac{(1 - \theta\beta)(1 - \theta)}{\theta},$$

and the marginal cost $\hat{mc}_t$ is

$$\hat{mc}_t = (\gamma + \gamma_n) \left(\hat{c}_t - \frac{1 + \gamma_n}{\gamma + \gamma_n} a_t \right) = (\gamma + \gamma_n) \left(\hat{y}_t - \frac{1 + \gamma_n}{\gamma + \gamma_n} a_t \right). \tag{57}$$

Or for simplicity one can define a new random variable $u_t^a = \kappa (1 + \gamma_n) a_t$ and rewrite equation (56) as

$$\pi_t = \kappa (\gamma + \gamma_n) \hat{c}_t + \beta E_t \pi_{t+1} - u_t^a. \tag{58}$$

The goods market equilibrium is still captured by the IS curve

$$\hat{c}_t = -\frac{1}{\gamma} \left[\hat{r}_t^n - E_t \pi_{t+1} \right] + E_t \hat{c}_{t+1}, \tag{59}$$

in which $\hat{r}_t^n$ is controlled by the monetary rule.

Now we can see that the most non-policy elements of the model are already captured in the IS curve (59) and new Keynesian Phillips curve (58), and such a two-equation system is often called a *new Keynesian model*.

Sometimes people express this system in terms of *output gap*. First let's think about how to rewrite equation (38) under flexible prices. Surely $\hat{s}_t = 0$ because there is no more price dispersion. And $\hat{mc}_t = 0$ because in this case the marginal cost is an exogenous constant for all the firms, as we argued in the last chapter. Therefore equation (38) becomes

$$\hat{y}_t^f - \gamma \hat{c}_t^f = (1 + \gamma_n) \hat{n}_t^f. \tag{60}$$

Also under flexible prices the form of resource constraint as well as production function remains the same,

$$\hat{c}_t^f = \hat{y}_t^f, \tag{61}$$

$$\hat{y}_t^f = a_t + \hat{n}_t^f. \tag{62}$$

Insert equations (61) and (62) into (60), and solve for $\hat{y}_t^f$

$$\hat{y}_t^f = \frac{1+\gamma_n}{\gamma+\gamma_n} a_t, \tag{63}$$

which is only determined by the exogenous shock a_t.

Apply equations (63) and (54) in (56) and express the new Keynesian Phillips curve in terms of aggregate output

$$\pi_t = \kappa (\gamma + \gamma_n) \left(\hat{y}_t - \hat{y}_t^f\right) + \beta E_t \pi_{t+1}. \tag{64}$$

Then we define $x_t = \hat{y}_t - \hat{y}_t^f$ as the gap between actual output level and the ideal output level when the prices were flexible, equation (64) becomes

$$\pi_t = \kappa (\gamma + \gamma_n) x_t + \beta E_t \pi_{t+1}. \tag{65}$$

Apply the fact, $x_t = \hat{y}_t - \hat{y}_t^f = \hat{c}_t - \frac{1+\gamma_n}{\gamma+\gamma_n} a_t$ on equation (59)

$$x_t + \frac{1+\gamma_n}{\gamma+\gamma_n} a_t = -\frac{1}{\gamma} \left[\hat{r}_t^n - E_t \pi_{t+1}\right] + E_t x_{t+1}. \tag{66}$$

Or for simplicity one can define a new random variable $\xi_t^a = \frac{1+\gamma_n}{\gamma+\gamma_n} a_t$ and rewrite equation (66) as

$$x_t = -\frac{1}{\gamma} \left[\hat{r}_t^n - E_t \pi_{t+1}\right] + E_t x_{t+1} - \xi_t^a. \tag{67}$$

An additional finding is that in this simplified model $\hat{mc}_t$, as in equation (57), is in fact equivalent to

$$\hat{mc}_t = (\gamma + \gamma_n) x_t, \tag{68}$$

i.e. the percentage deviation of the real marginal cost from its steady state level is proportional to the output gap. Remember that when a firm z asks price $P_t(z)$ for its product, its mark-up can be written as

$$1 + \mu_t(z) = \frac{P_t(z)}{P_t MC_t}. \tag{69}$$

Surely with rigidities in price adjustment $\mu_t(z)$ cannot be the same for all the firms, since $P_t(z)$s are dispersed on both sides of P_t. However we can write

$$1 + \mu_t = \frac{1}{MC_t} \tag{70}$$

to measure the *average* markup in each period t. Log-linearize equation (70) and we find that

$$\hat{\mu}_t = -\hat{mc}_t = -(\gamma + \gamma_n)\, x_t, \tag{71}$$

meaning that the firms' average markup is counter-cyclical.

Now, alternatively, in terms of output gap the system is characterized by equations (65) and (67). In the next two sections, we stick to the former specification, i.e. the system of equations (58) and (59).

4.2 Monetary Rule: Feedback Rules versus Non-Feedback Rules

Now in this simplified economy the general equilibrium is captured by equations (56) and (59), containing three variables: the consumption gap $\hat{c}_t$ (equivalent to the output gap $\hat{y}_t$ in this economy), inflation π_t and the nonimal interest rate $\hat{r}^n_t$. The model would be closed if there exists any equation characterizing $\hat{r}^n_t$, suggesting there is a role for monetary rule [3] such as (48) here to explicitly define the path of $\hat{r}^n_t$.

Rule (48) is just an example, and surely there is an infinite number of candidates for monetary rules. Here we consider two types of the rules which may be interesting.

4.2.1 Non-Feedback Rules

One type of the rules could be called *non-feedback rules*. Suppose that the central bank feels that some certain level of $\hat{r}^n_t$, for example, the natural rate, works well for the economy. Then the central bank wants to follow such "best practice"by defining the following $AR(1)$ process as its monetary rule

$$\hat{r}^n_t = \rho^r \hat{r}^n_{t-1} + \upsilon^r_t, \tag{72}$$

in which the random variable υ^r_t is a monetary innovation in each period being not correlated with $\hat{c}_t$ and π_t. Will this rule work?

[3] Of course the central bank can pick up an endogenous equation (51) as its monetary rule, such that it defines the interest rate by adjusting money supply. However, in our model what we want to explain is that monetary shocks have real effects, and this is clearly unrelated to any money demand considerations. Therefore we would like to have a *cashless* economy à la Woodford (2003), and this is why we specify an interest rate rule here. Although money supply is always implicitly satisfied via equation (51) after the path of nominal interest rate is given, it is no longer interesting in our model.

To examine this, notice that now the system can be expressed as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \gamma \\ 0 & \beta & 0 \end{bmatrix} \begin{bmatrix} \hat{r}^n_t \\ E_t\pi_{t+1} \\ E_t\hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho^r & 0 & 0 \\ \rho^r & 0 & \gamma \\ 0 & 1 & -\kappa(\gamma+\gamma_n) \end{bmatrix} \begin{bmatrix} \hat{r}^n_{t-1} \\ \pi_t \\ \hat{c}_t \end{bmatrix} + \begin{bmatrix} \upsilon^r_t \\ \upsilon^r_t \\ u^a_t \end{bmatrix} \tag{73}$$

by combining equations (56), (59) and (72), then rearrange to get

$$\begin{bmatrix} \hat{r}^n_t \\ E_t\pi_{t+1} \\ E_t\hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho^r & 0 & 0 \\ 0 & \beta^{-1} & -\frac{\kappa(\gamma+\gamma_n)}{\beta} \\ \frac{\rho^r}{\gamma} & -\frac{1}{\gamma\beta} & 1+\frac{\kappa(\gamma+\gamma_n)}{\gamma\beta} \end{bmatrix} \begin{bmatrix} \hat{r}^n_{t-1} \\ \pi_t \\ \hat{c}_t \end{bmatrix} + \begin{bmatrix} \upsilon^r_t \\ \frac{u^a_t}{\beta} \\ \frac{\upsilon^r_t}{\gamma} - \frac{u^a_t}{\gamma\beta} \end{bmatrix}. \tag{74}$$

Again we take REE (rational expectation equilibrium) as our equilibrium selection criteria. As is seen in Chapter 7, since $\hat{c}_t$ and π_t are *both* endogenous variables (note that in the model of Chapter 7 only 2 out of 4 variables were endogenous), the system is determinant for a unique, stationary solution if and only if *both* eigenvalues of the following partition

$$\begin{bmatrix} \beta^{-1} & -\frac{\kappa(\gamma+\gamma_n)}{\beta} \\ -\frac{1}{\gamma\beta} & 1+\frac{\kappa(\gamma+\gamma_n)}{\gamma\beta} \end{bmatrix}$$

are larger than 1 (i.e. "outside the unit circle"). However, with some simple algebra one can see that

$$\begin{aligned} \lambda_1 &= \frac{\gamma\beta+\kappa(\gamma+\gamma_n)+\gamma+\sqrt{\gamma^2\beta^2+2\gamma\beta\kappa(\gamma+\gamma_n)-2\gamma^2\beta+(\kappa(\gamma+\gamma_n))^2+2\kappa(\gamma+\gamma_n)\gamma+\gamma^2}}{2\gamma\beta} \\ &> 1, \text{ but} \\ \lambda_2 &= \frac{\gamma\beta+\kappa(\gamma+\gamma_n)+\gamma-\sqrt{\gamma^2\beta^2+2\gamma\beta\kappa(\gamma+\gamma_n)-2\gamma^2\beta+(\kappa(\gamma+\gamma_n))^2+2\kappa(\gamma+\gamma_n)\gamma+\gamma^2}}{2\gamma\beta} \\ &< 1! \end{aligned}$$

This implies that multiple equilibria exist and the system is indeterminate. To see this, suppose that for whatever reason there comes a rise in expected inflation. Since the monetary rule (72) doesn't make any response to such change in expectation, the real interest rate has to fall due to Fisher's parity. But then the fall in real interest rate increases the output gap, as is seen from IS curve, and this eventually leads to an actual inflation, as is seen from new Keynesian Phillips curve. Therefore, just change in expectation, which is non-fundamental, results a self-fulfilling change in actual inflation, and the equilibrium path of the system is indeterminant.

4.2.2 *Feedback Rules*

The alternative rule could be that the central bank responds the endogenous variables. To make it as simple as possible, consider the following monetary rule

$$\hat{r}_t^n = \gamma_\pi \pi_t + \epsilon_t^r \tag{75}$$

in which the nominal interest rate $\hat{r}_t^n$ is adjusted to trace the observed inflation rate π_t, and the random variable ϵ_t^r is a white noise.

Now merge equation (75) with (59), and combine with (56), the system can be expressed as

$$\mathbf{E}_t \begin{bmatrix} \pi_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa(\gamma+\gamma_n)}{\beta} \\ \frac{\gamma_\pi - \frac{1}{\beta}}{\gamma} & 1 + \frac{\kappa(\gamma+\gamma_n)}{\beta\gamma} \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{c}_t \end{bmatrix} + \begin{bmatrix} u_t^a \\ \frac{\epsilon_t^r}{\gamma} \end{bmatrix}. \tag{76}$$

Whether the system is determinant or not depends on eigenvalues of the coefficient matrix

$$\begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa(\gamma+\gamma_n)}{\beta} \\ \frac{\gamma_\pi - \frac{1}{\beta}}{\gamma} & 1 + \frac{\kappa(\gamma+\gamma_n)}{\beta\gamma} \end{bmatrix}, \tag{77}$$

which can be computed by some tedious algebra,

$$\lambda_1 = \frac{\beta^2\gamma + \beta\kappa(\gamma+\gamma_n) + \gamma\beta + \sqrt{\beta^4\gamma^2 + 2\beta^3\gamma\kappa(\gamma+\gamma_n) - 2\beta^3\gamma^2 + \beta^2(\kappa(\gamma+\gamma_n))^2 - 2\beta^2\kappa(\gamma+\gamma_n)\gamma + \gamma^2\beta^2 - 4\gamma\beta^2\kappa(\gamma+\gamma_n)\gamma_\pi + 4\gamma\beta\kappa(\gamma+\gamma_n)}}{2\beta^2\gamma},$$
$$\lambda_2 = \frac{\beta^2\gamma + \beta\kappa(\gamma+\gamma_n) + \gamma\beta - \sqrt{\beta^4\gamma^2 + 2\beta^3\gamma\kappa(\gamma+\gamma_n) - 2\beta^3\gamma^2 + \beta^2(\kappa(\gamma+\gamma_n))^2 - 2\beta^2\kappa(\gamma+\gamma_n)\gamma + \gamma^2\beta^2 - 4\gamma\beta^2\kappa(\gamma+\gamma_n)\gamma_\pi + 4\gamma\beta\kappa(\gamma+\gamma_n)}}{2\beta^2\gamma}.$$

However, the values still depend on γ_π, i.e. how the central bank specify its monetary rule. To make both of them larger than 1, it's sufficient to ensure that the smaller one,

$$\lambda_2 > 1,$$

which leads to a rather simple condition (although the algebra is tedious)

$$\begin{aligned} -\beta^2 + \beta + \beta\gamma_\pi - 1 > 0, & \\ \gamma_\pi > & \frac{\beta^2 - \beta + 1}{\beta} \\ = & \beta + \frac{1}{\beta} - 1 \\ \geq & 1, \end{aligned}$$

$\forall\beta \in (0, +\infty)$, and the equality only holds when $\beta = 1$. Usually β is just several percent less than 1, and $\frac{\beta^2-\beta+1}{\beta}$ is nearly 1, therefore the central bank should specify its monetary rule with a sufficiently large feedback to inflation rate, $\gamma_\pi > 1$, i.e. the central bank should be aggressive in the response to inflation — This is often referred as the *Taylor Principle* (Taylor, 1993).

This simple exercise tells us that the feedback rules may be desirable for central banks, given that a proper γ_π is chosen. The interesting thing is, if Taylor Principle holds, then in equilibrium π_t and $\hat{c}_t$ will be exactly zero, i.e. people would never deviate and the central bank's aggressive response never materialize. The reason is, if π_t and $\hat{c}_t$ are non-zero in equilibrium, the future inflation and output gap will gradually go to infinity because both eigenvalues of (77) are outside the unit circle! Knowing this, the rational agents would never make the explosion happen in the first place, i.e. if the central bank is credible in the sense that it would definitely explode the world whenever the agents deviate from the equilibrium, then the equilibrium is determinant!

4.3 Simplification versus Complication

The discussions in Section 4.1 and 4.2 depend on a much simplified version of our original model by dropping off fiscal shocks and capital accumulation procedure, which is nearly the same as the seminal Calvo-Yun (Yun, 1996) model and already becomes a standard textbook treatment of introducing new Keynesian economics, for example, Gertler (2003), Galí (2008) and so on. What's more, it is already seen that even this simplified model is pretty successful in explaining the economic dynamics and gives a desired, clear-cut principle in designing monetary rules. Then readers may wonder why we are making such a big fuss of complications in these two chapters rather than starting directly from the simplified economy as the other authors do — If we compare with the models in physics, most textbook models are established without considering frictions because one can easily extend these models to include frictions without having to introduce new *mechanisms* beyond the Laws of Newton — So why need people complicate the seminal Calvo-Yun framework by adding new transmission mechanisms (fiscal shocks, capital accumulation procedure, and so on) instead of sticking to (or just extending) those in the original model (featured by monopolistic competition and price stickiness)?

The failure of Calvo-Yun model can be easily seen in the following. Suppose, as a simplest case, monetary shock is the only shock[4] in the economy and the system is made determinant by some approapriate γ_π. So the system is characterized by equation (76) with $u_t^a = 0$,

$$\mathbf{E}_t \begin{bmatrix} \pi_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa(\gamma+\gamma_n)}{\beta} \\ \frac{\gamma_\pi - \frac{1}{\beta}}{\gamma} & 1 + \frac{\kappa(\gamma+\gamma_n)}{\beta\gamma} \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{c}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\epsilon_t^r}{\gamma} \end{bmatrix}.$$

[4] The reason is that we we want to isolate monetary shock from the others, in order to see exactly how the system reacts.

Then it's immediately seen that the rational expectation equilibrium requires that

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa(\gamma+\gamma_n)}{\beta} \\ \frac{\gamma_\pi - \frac{1}{\beta}}{\gamma} & 1 + \frac{\kappa(\gamma+\gamma_n)}{\beta\gamma} \end{bmatrix} \begin{bmatrix} \pi_0 \\ \hat{c}_0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\epsilon_0^r}{\gamma} \end{bmatrix}, \tag{78}$$

in which π_0 and $\hat{c}_0$ replaced π_t and $\hat{c}_t$ because of the *timeless perspective* of REE, i.e. the endogenous variables should be determinant in any *period*. Now equation (78) is just a system of two equations containing two unknowns, therefore we can easily solve

$$\begin{aligned} \pi_0 &= -\frac{\kappa(\gamma+\gamma_n)}{\gamma_\pi\kappa(\gamma+\gamma_n)+\gamma}\epsilon_0^r \\ \hat{c}_0 &= -\frac{1}{\gamma_\pi\kappa(\gamma+\gamma_n)+\gamma}\epsilon_0^r, \end{aligned}$$

and the percentage change in nominal interest rate is determined as

$$\begin{aligned} \hat{r}_0^n &= \gamma_\pi\pi_0 + \epsilon_0^r \\ &= -\frac{\gamma_\pi\kappa(\gamma+\gamma_n)}{\gamma_\pi\kappa(\gamma+\gamma_n)+\gamma}\epsilon_0^r + \epsilon_0^r \\ &= \frac{\gamma}{\gamma_\pi\kappa(\gamma+\gamma_n)+\gamma}\epsilon_0^r. \end{aligned}$$

Then for a postive shock ϵ_0^r, i.e. a contraction in monetary policy, it's easily seen that $\hat{r}_0^n > 0$, $\pi_0 < 0$ and $\hat{c}_0 < 0$ — The simplified model makes right predictions in the impulse responses. However, it's also immediately seen that the simplified model predicts that a monetary shock has only *one-period* effect, which contradicts to the evidences (e.g. Christiano, Eichenbaum and Evans, 2005) that

- effect of monetary policy (isolated from the other shocks) is usually delayed and highly persistent; and
- inflation peaks only *after* output has peaked.

Therefore the simplified model is a poor explanation of the *persistence* in the observed effects of monetary shocks. So that's why current works tend to include real rigidities, such as capital adjustment cost in our model, whose effect persist for multiple periods, hoping to replicate more facts in reality.

4.4 Optimal Fiscal / Monetary Policy

First we have to keep in mind that what we have done so far is to charaterize the equilibrium under some given policy, rather than to discuss which kind of rules would make the economy better off. The latter is so big a question that deserves a separate chapter.

However, since we have got a complete understanding of the economy, it's possible now to think where the inefficiencies are and how to eliminate them by policy design. Suppose that a benevolent social planner seeks to maximize the representative household's welfare, given technology, preferences and resources via some *Ramsey policy*. Then here may be some misallocations in our economy that the social planner wants to correct:

- The distortion related to monopolistic competition. Section 3.1.1 of Chapter 9 already shows that monopolistic competition distorts factor prices, hence the agent's intratemporal decisions on consumption and labor,

  $$-\frac{\frac{\partial u_t}{\partial N_t}}{\frac{\partial u_t}{\partial C_t}} = \frac{W_t}{P_t} = \frac{1}{1+\mu}\frac{\partial Y(z)}{\partial N(z)}.$$

  This suggests that it would be optimal to subsidize the employment cost. Suppose that at the rate τ the employment is subsidized, then

  $$-\frac{\frac{\partial u_t}{\partial N_t}}{\frac{\partial u_t}{\partial C_t}} = (1+\tau)\frac{W_t}{P_t} = \frac{1+\tau}{1+\mu}\frac{\partial Y(z)}{\partial N(z)}$$

  and the optimality would be restored if the social planner sets $\tau = \mu$;
- The distortion related to staggering price adjustments, which is shown in Section 2.1.2. The output level, i.e. the production of the final goods, is distorted by the factor of s_t due to the existence of price dispersion. Since the prices are adjusted in a staggering manner in our economy, the only way to wipe out such inefficient price dispersion is to keep the price level constant, such that the firms don't have to adjust their prices at all. Therefore, it would a desired policy to stablize the price level, i.e. to eliminate inflation, in this economy;
- However, note that in our model the money demand comes from the money-in-the-utility setups. Then Friedman Rule reminds us that the first-best solution for money holdings should be achieved under a *deflationary* monetary policy, which seems to be conflicting with the former requirement for price stablization.

Except to discuss *where* to introduce the policies, anther question is *how* to design and implement the policies. For example,

- What parameters should the policy respond? As we have examined, at least in our simplified model, the feedback rules might be desirable;
- Should policies be committed to? In our model whenever we introduce monetary rules we implicitly assume that the central bank sticks to its published rule forever. However, it can be shown that the central bank may get some short-term gain if it deviates from the rule, surprising the agents in the economy.

These issues are left for future lectures and exercises.

5 Readings

Clarida, Galí and Gertler (1999); Galí (2008), CHAPTER 3.

6 Bibliographic Notes

The seminal general equilibrium based new Keynesian model is Yun (1996), soon followed by Goodfriend and King (1997) and Rotemberg and Woodford (1997). Clarida, Galí and Gertler (1999) is a standard reference for understanding the *new Keynesian perspective*.

Christiano, Eichenbaum and Evans (2005) characterized the dynamic effects of a monetary shock using VAR approach, and soon becomes a standard reference for people seeking to capture the *persistence*. Currently much effort is exerted on this issue (most of the works are combined with seeking optimal policies given certain sources of persistence), and quite a few ideas have been proposed. Altig, Christiano, Eichenbaum and Linde (2005), Sveen and Weinke (2005, 2007) extend the idea of Woodford (2003, CHAPTER 5) by assuming that the capital is firm-specific. Ravn, Schmitt-Grohé and Uribe (2006) extends habit persistence in monopolistic competition such that consumers have *deep* habit on all differentiated goods. Schmitt-Grohé and Uribe (2005) integrates a rich array of real and nominal rigidities that have been identified in the recent literature (four nominal frictions: sticky prices, sticky wages, demand for money by households, and a cash-in-advance constraint on the wage bill of firms, and five sources of real rigidities: investment adjustment costs, variable capacity utilization, habit formation, imperfect competition in product and factor markets, and distortionary taxation) in a single general equilibrium model.

Although Taylor principle has been widely accepted for almost two decades, it is now under the fatal attack by Cochrane (2011).

7 Exercises

7.1 Sticky Price Models: Implications

Consider the models with sticky prices, such that some firms don't make immediate responses to the changes in the price level (Surely Calvo-Yun model is one of them).

a)[A] Explain the difference between *ex ante* and *ex post* mark-up.

b)[A] Explain how monetary policy affects the real economy.

7.2 Calvo-Yun Model: How Everything Works

Consider the Calvo-Yun model with staggered price setting.

a)^{A} Show that increases in output have a positive impact on inflation.

b)^{A} Explain, why the resulting aggregate supply curve is forward looking.

c)^{C} Explain how the economy is distorted by monopolistic competition and staggered price setting. Provide some intuitions on how economic policies may restore the efficiency of equilibrium allocations.

d)^{B} Show that stabilization of output and inflation are no conflicting goals.

References

Altig, D., L. J. Christiano, M. Eichenbaum and J. Linde (2005): "Firm -Specific Capital, Nominal Rigidities and the Business Cycle."National Bureau of Economic Research Working Paper No. 11034.

Christiano, L. J., M. Eichenbaum and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy."*Journal of Political Economy*, 113, Feburary, 1–45.

Clarida, R., J. Galí and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective."*Journal of Economic Literature*, 37, December, 1661–1707.

Cochrane, J. H. (2011): "Determinacy and Identification with Taylor Rules."*Journal of Political Economy* (*forthcoming*).

Debreu, G. (1959): *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*. New Haven: Yale University Press.

Galí, J. (2008): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton: Princeton University Press.

Gertler, M. (2003): "A Lecture on Sticky Price Models."Mimeo, New York University.

Goodfriend, M. and R. G. King (1997): "The New Neoclassical Synthesis and the Role of Monetary Policy."In *NBER Macroeconomics Annual 1997*, by B. S. Bernanke and J. J. Rotemberg (Eds.), 231–283. Cambridge: MIT Press.

Levin, A. T., A. Onatski, J. C. Williams and N. Williams (2005): "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models."In *NBER Macroeconomics Annual 2005*, by M. Gertler and K. Rogoff (Eds.), 229–287. Cambridge: MIT Press.

Rotemberg, J. J. and M. Woodford (1997): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy."In *NBER Macroeconomics Annual 1997*, by B. S. Bernanke and J. J. Rotemberg (Eds.), 297–346. Cambridge: MIT Press.

Ravn, M., S. Schmitt-Grohé and M. Uribe (2006): "Deep Habits."*Review of Economic Studies*, 73, January, 195–218.

Romer, D. (2006): *Advanced Macroeconomics (3rd Ed.)*. Boston: McGraw-Hill Irwin.

Schmitt-Grohé, S. and M. Uribe (2005): “Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model.”In: Gertler, M. and K. Rogoff, (Eds.), *NBER Macroeconomics Annual 2005*. Cambridge: MIT Press. 383–425.

Sveen, T. and L. Weinke (2005): “New Perspectives on Capital, Sticky Prices, and the Taylor Principle.”*Journal of Economic Theory*, 123, July, 21–39.

Sveen, T. and L. Weinke (2007): “Firm-Specific Capital, Nominal Rigidities, and the Taylor Principle.”*Journal of Economic Theory*, 136, September, 729–737.

Taylor, J. B. (1993): “Discretion versus Policy Rules in Practice.”*Carnegie-Rochester Conference Series on Public Policy*, 39, December, 195–214.

Walsh, C. E. (2010): *Monetary Theory and Policy (3rd Ed.)*. Cambridge: MIT Press.

Woodford, M. (2003): *Interest and Prices*. Princeton: Princeton University Press.

Yun, T. (1996): “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles.”*Journal of Monetary Economics*, 37, April, 345–370.

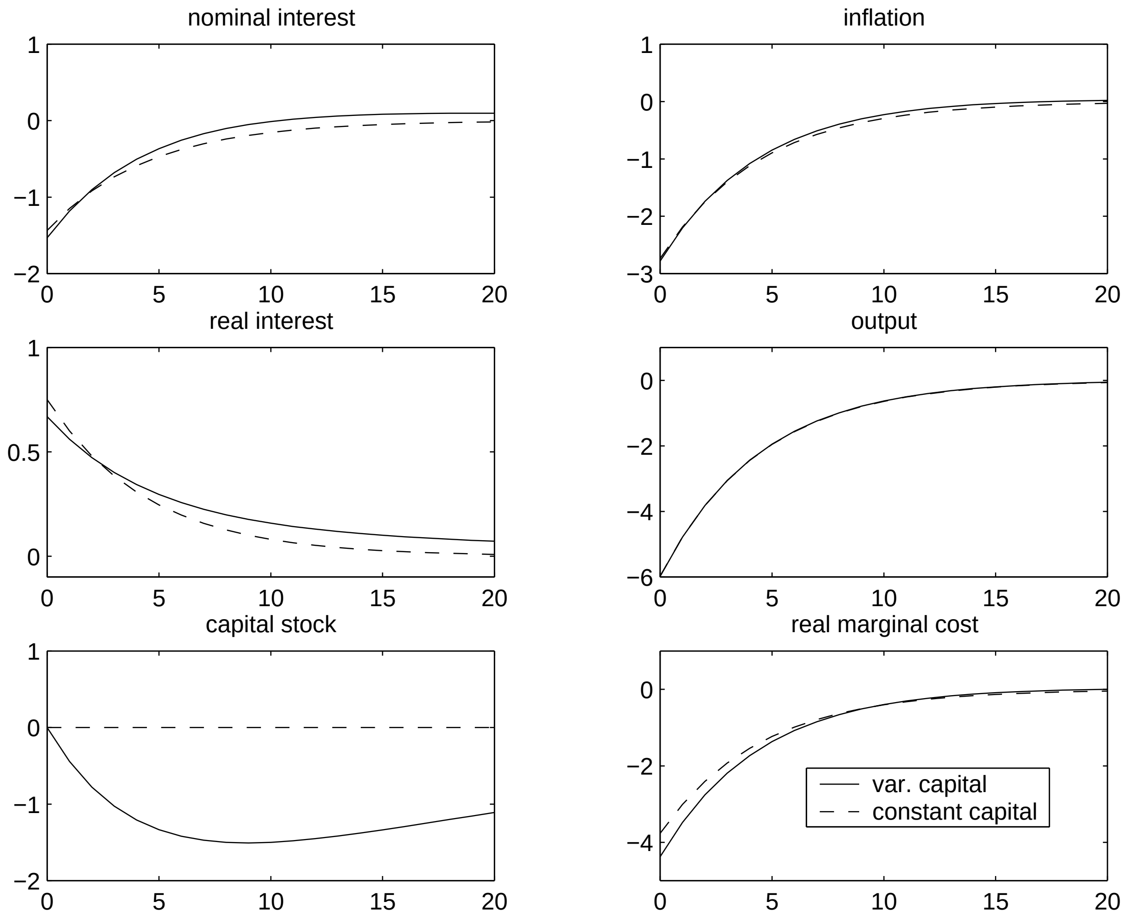

Fig. 1. Impulse Responses to an Unexpected Monetary Tightening: Constant-Capital Model versus Variable-Capital Model (Woodford, 2003)